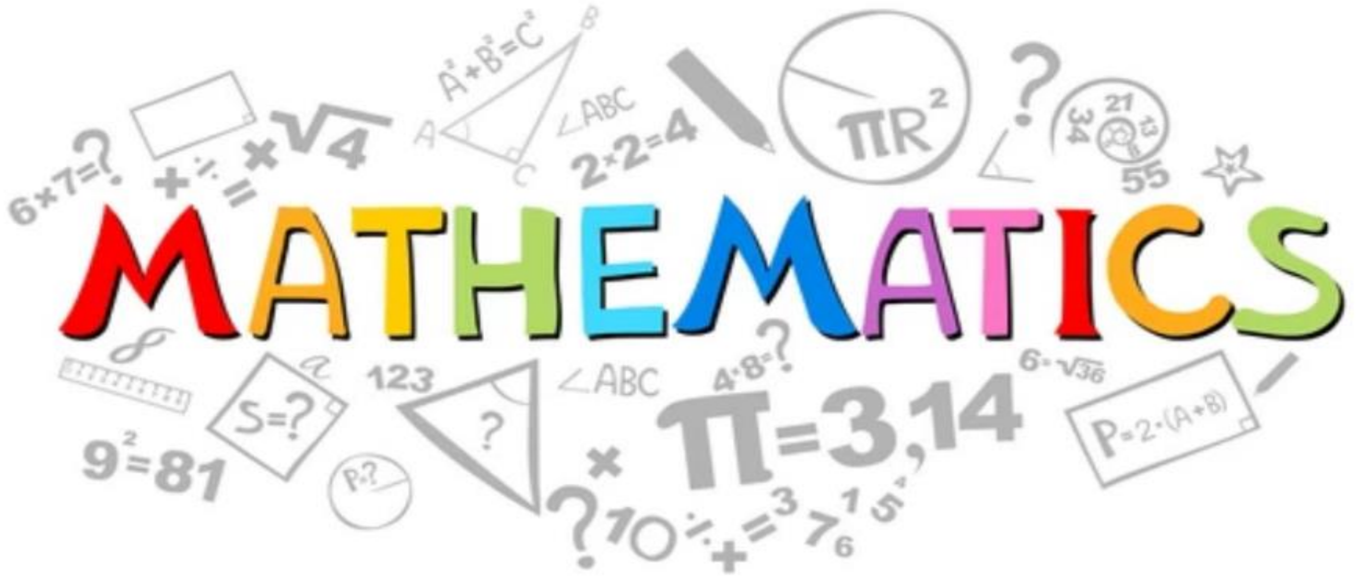
केन्द्रीय विद्यालय संगठन
क्षेत्रीय कार्यालय राँची
KENDRIYA VIDYALAYA SANGATHAN
RANCHI REGION

STUDENT SUPPORT MATERIAL
SESSION: 2023-24

तत् त्वं पूषन् अपावृणु
केन्द्रीय विद्यालय संगठन

CLASS X



Our Patron

Shri. D.P. Patel
Deputy Commissioner
KVS RO Ranchi

Shri. SURESH KUMAR SINGH
Assistant Commissioner
KVS RO Ranchi

SMT. SUJATA MISHRA
Assistant Commissioner
KVS RO Ranchi

Shri. BALENDER KUMAR
Assistant Commissioner
KVS RO Ranchi

Shri. SOMIT KUMAR
CONVENOR
PRINCIPAL KV, NAMKUM

Shri. ROHIT KUMAR
ASSOCIATE CONVENOR
TGT-MATHS, KV NAMKUM

मेरे प्रिय विद्यार्थियों के लिए दो शब्द

केन्द्रीय विद्यालय संगठन, राँची संभाग के 10वीं कक्षा के विद्यार्थियों हेतु छात्र सहायता सामग्री प्रस्तुत करते हुए मुझे अपार हर्ष हो रहा है। 10वीं कक्षा के छात्रों, यह सामग्री आपकी आवश्यकताओं को ध्यान में रखते हुए तैयार की गई है। जब आप परीक्षा की तैयारी के अंतिम चरण में होते हैं तब आप एक स्थान पर सभी संभावित प्रश्नों को देख अपने श्रम को एकाग्र कर पाते हैं। इस सहायता सामग्री से प्रश्नों को दोहराना और अभ्यास करना सहज होगा। जब आप आवंटित समय में प्रश्न पत्र पूरा करने की अपनी क्षमता का परीक्षण करना चाहते हों या जब आप अध्ययन करते समय कोई प्रश्न देखते हों तब यह सहायक सामग्री आपकी सहायता हेतु होगा। तैयारी करते समय कभी-कभी तत्काल उत्तर की आवश्यकता है, लेकिन पाठ्य-पुस्तक से खोजने और पढ़ने में समय लगेगा। वैसी स्थिति में जब आप कम समय में पूरी अवधारणा या विचार को समझना और जानना चाहते हैं तब यह सामग्री आपको तुरंत परेशानी से बचा लेगी। यह पिछले सीबीएसई बोर्ड परीक्षा के पेपर और प्रतियोगी परीक्षा के किसी प्रश्न को जानने और समझने में मदद करेगा।

अपने विषयों में विशेषज्ञता रखने वाले समर्पित और अनुभवी शिक्षकों की एक टीम ने कड़ी मेहनत के बाद इस सामग्री को तैयार किया है। केवल उन्हीं वस्तुओं को शामिल करने का ध्यान रखा गया है जो प्रासंगिक हैं और पाठ्य-पुस्तक के अनुरूप हैं। इस सामग्री को एनसीईआरटी पाठ्य पुस्तक के विकल्प के रूप में नहीं लिया जाना चाहिए बल्कि इसे इसके पूरक के रूप में डिज़ाइन किया गया है। छात्रों की सहायता सामग्री में आपके लिए आवश्यक सभी महत्वपूर्ण पहलू हैं: प्रश्न पत्र का डिज़ाइन, पाठ्यक्रम, सभी इकाइयों/अध्यायों या बिंदुओं में अवधारणाएं, प्रत्येक अध्याय से नमूना परीक्षण आदि । मुझे यकीन है कि सहायक सामग्री का उपयोग छात्रों और शिक्षकों दोनों द्वारा किया जाएगा और मुझे विश्वास है कि यह सामग्री आपको अपनी परीक्षाओं में अच्छा प्रदर्शन करने में मदद करेगी । आनेवाली परीक्षा के लिए शुभकामनाओं के साथ आप यह अवश्य याद रखें मेहनत का कोई विकल्प नहीं है।

डी.पी.पटेल

उपायुक्त

केन्द्रीय विद्यालय संगठन

क्षेत्रीय कार्यालय राँची

KENDRIYA VIDYALAYA SANGATHAN
RANCHI REGION

TEAM OF TEACHERS WHO CONTRIBUTED

I. STUDY MATERIAL PREPARATION TEAM (Shri./Smt.):

SN	NAME OF TEACHER	DESIGNATION	NAME OF KV	CHAPTER PREPARED
1	SARITA KUMARI	TGT MATHS	GARWAH	1. REAL NUMBERS
2	RANJANA KUMARI	TGT MATHS	CHAKRADHARPUR	2. POLYNOMIALS
3	ABHIRAM BANRA	TGT MATHS	MEGAHATUBURU	3. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES
4	ROOPSHREE	TGT MATHS	CRPF RANCHI	4. QUADRATIC EQUATIONS
5	RITESH RANJAN	TGT MATHS	HINOO 1ST SHIFT	5. ARITHMETIC PROGRESSION
6	JAYANT KUMAR PATAR	TGT MATHS	MAITHAN DAM	6. TRIANGLES
7	PREETY	TGT MATHS	BHURKUNDA	7. COORDINATE GEOMETRY
8	DEEPAK SINGH BORA	TGT MATHS	SINGHARSHI	8. INTRODUCTION TO TRIGONOMETRY
9	ASHOK KUMAR MAHATO	TGT MATHS	HINOO 2ND SHIFT	9. SOME APPLICATION OF TRIGONOMETRY
10	KASHINATH MAHAPATRA	TGT MATHS	CHAKRADHARPUR	10. CIRCLE
11	BERNARD I PURTY	TGT MATHS	LATEHAR	12. AREA RELATED TO CIRCLE
12	MANISHA JAISWAL	TGT MATHS	CCL RANCHI	13. SURFACE AREA AND VOLUME
13	PUJA KUMARI	TGT MATHS	PATRATU	14. STATISTICS
14	SONALI SINGH	TGT MATHS	Ramgarh Cant	15.PROBABLITY

II. MODERATION TEAM (Shri/Smt):

SN	NAME OF TEACHER	DESIGNATION	NAME OF KV	CHAPTERS MODERATED/RECHECKED
1	PARMANAND RANA	TGT MATHS	CRPF Ranchi	1. REAL NUMBERS 2. POLYNOMIALS
2	SANDHYA GUPTA	TGT MATHS	Tata Nagar	3. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES 5. ARITHMETIC PROGRESSION
3	FAIZ AHMAD	TGT MATHS	Ramgarh Cant	6. TRIANGLES 7. COORDINATE GEOMETRY
4	AJAY KUMAR	TGT MATHS	Hinoo 1st Shift	8. INTRODUCTION TO TRIGONOMETRY 9. SOME APPLICATION OF TRIGONOMETRY
5	SUJATA NAYAK	TGT MATHS	Ramgarh Cant	12. AREA RELATED TO CIRCLE 13. SURFACE AREA AND VOLUME
6	JITENDRA KUMAR	TGT MATHS	Megahatuburu	14. STATISTICS 15.PROBABLITY
7	ABHISHEK DUBEY	TGT MATHS	Dipatoli	4. QUADRATIC EQUATIONS 10. CIRCLE

III. FINAL EDITING & COMPILATION :

1.	Shri ROHIT KUMAR	TGT MATHS	KV NAMKUM

CONTENT INDEX

important,   ** very important,   *** most important

MATHEMATICS (CODE NO. 041/241) COURSE STRUCTURE

CLASS –X

Units	Unit Name	Marks
I	NUMBER SYSTEMS	06
II	ALGEBRA	20
III	COORDINATE GEOMETRY	06
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILTY	11
	TOTAL	80

UNIT I: NUMBER SYSTEMS

1. REAL NUMBER (15) Periods

Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}$, $\sqrt{3}, \sqrt{5}$

UNIT II: ALGEBRA

1. POLYNOMIALS (8) Periods

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES (15) Periods

Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency. Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS (15) Periods

Standard form of a quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS (10) Periods

Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY

Coordinate Geometry (15) Periods

Review: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division).

UNIT IV: GEOMETRY

1. TRIANGLES (15) Periods

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.

3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.

4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.

5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES (10) Periods

Tangent to a circle at, point of contact

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.

2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

8 UNIT V: TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY (10) Periods

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0^o and 90^o. Values of the trigonometric ratios of 30^0 , 45^0 and 60^0 . Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES (15) Periods

Proof and applications of the identity $\sin^2A + \cos^2A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES:

Angle of elevation, Angle of Depression. (10) Periods

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, and 60°.

UNIT VI: MENSURATION

1. AREAS RELATED TO CIRCLES (12) Periods

Area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only.

2. SURFACE AREAS AND VOLUMES (12) Periods

Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones.

UNIT VII: STATISTICS AND PROBABILITY

1. STATISTICS (18) Periods

Mean, median and mode of grouped data (bimodal situation to be avoided).

2. PROBABILITY (10) Periods

Classical definition of probability. Simple problems on finding the probability of an event.

MATHEMATICS-Standard (041)

QUESTION PAPER DESIGN

CLASS – X (2023-24)

Time: 3 Hours Max **Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1.	**Remembering**: Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	**43**	**54**
2.	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	**19**	**24**
3.	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	**18**	**22**
4.	**Total**	**80**	**100**

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

MATHEMATICS-Basic (241)

QUESTION PAPER DESIGN

CLASS – X (2023-24)

Time: 3 Hours Max **Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1.	**Remembering**: Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	**60**	**75**
2.	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	**12**	**15**
3.	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	**08**	**10**
4.	**Total**	**80**	**100**

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

DELETED TOPICS

CHAPTER NAME	PAGE NUMBERS	DROPPED TOPICS
REAL NUMBERS	2-7 , 15-18	1.2 EUCLID'S DIVISION LEMMA 1.5 REVISITING RATIONAL NUMBERS AND THEIR DECIMAL EXPANSION
POLYNOMIALS	33-37	DIVISION OF ALGORITHM FOR POLYNOMIALS
PAIR OF LINEAR EQUATIONS IN TWO VARIABLES	PAGE 67	3.4 CROSS MULTIPLICATION METHOD (ONLY METHOD, NOT QUESTIONS) 3.5 EQUATIONS REDUCIBLE TO A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES. EXERCISE 3.6 COMPLETELY DELETED
QUADRATIC EQUATIONS	77 - 82	4.4 SOLUTION OF A QUADRATIC EQUATION BY COMPLETING THE SQUARE (ONLY METHOD IS DELETED)
ARITHMETIC PROGRESSION	--------	NO DELETION
TRIANGLES	--------	Theorem 6.2 converse of BPT Theorem 6.3 Theorem 6.4 Theorem 6.5 Exercise 6.4 completely DELETED Theorem 6.7, Pythagoras theorem 6.8, 6.9 CONVERSE Exercise 6.5 completely deleted
COORDINATE GEOMETRY	168 - 170	AREA OF A TRIANGLE EXERCISE 7.3 DELETED
INTRODUCTION TO TRIGONOMETRY	187 - 190	TRIGONOMETRIC RATIOS OF COMPLEMENTARY ANGLES 8.2 DELETED 8.3 DELETED 8.4 (QN NO 3)

APPLICATIONS OF TRIGONOMETRY	--------	NO DELETION
CIRCLES	--------	NO DELETION
AREA RELATED TO CIRCLES	231 - 238	PLANE FIGURES INVOLVING TRIANGLES, SIMPLE QUADRILATERALS AND CIRCLE(COMBINATION OF PLANE FIGURES) IN CALCULATING AREA OF A SEGMENTOF A CIRCLE PROBLEMS SHOULD BE RESTRICTED TO THE CENTRAL ANGLES OF 60º 90º AND 120º ONLY. EXAMPLE 2 IN PAGE 228 IS DELETED EXERCISE 12.2(QNS 11 & 12) DELETED EXERCISE12.3 DELETED,
SURFACE AREAS AND VOLUMES	247, 249 - 258	Example 7 in page no 247 deleted In 13.2 QUESTIONS 5 and 7 deleted CONVERSION OF SOLID FROM ONE SHAPE TO ANOTHER EXERCISE 13.3 COMPLETELY DELETED FRUSTUM OF A CONE DELETED AND RELATED EXERCISE 13.4 DELETED
STATISTICS	289-291	GRAPHICAL REPRESENTATION OF CUMULATIVE FREQUENCY DISTRIBUTION EXERCISE 14.4 DELETED
PROBABILITY	--------	NO DELETION

CHAPTER - 1

REAL NUMBERS

Prepared by- SARITA KUMARI,
TGT (MATHS)
KV GARHWA

KEY CONCEPTS AND FORMULAE

- THE FUNDAMENTAL THEOREM OF ARITHMETIC: Every composite number can be expressed as a product of primes, and this factorization is unique except for the order in which the prime factors occur.
- Every composite number can be uniquely expressed as the product of powers of primes in ascending or descending order.
- For any two positive integers a and b, HCF (a, b) x LCM(a, b) = a x b.
- Let 'a' be a positive prime number such that 'p' is divisible by a^2, then p is also divisible by a.
- There are infinitely many positive primes.
- If p is a positive prime, then $\sqrt{p}$ is an irrational number. For example, $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{7}$, $\sqrt{11}$ etc. are irrational numbers.

SOLVED EXAMPLES

EXAMPLE 1: Show that 12^n cannot end with digit 0 or 5 for any natural number n.

SOLUTION: Expressing 12 as the product of primes, we obtain

$$12 = 2^2 \times 3,$$

$$12^n = (2^2 \times 3)^n = (2^2)^n \times 3^n = (2)^{2n} \times 3^n$$

So, only primes in the factorization of 12^n are 2 and 3 and, not 5. Hence, 12 cannot end with digit 0 or 5.

EXAMPLE 2: Find the HCF of 96 and 404 by prime factorization method. Hence, find their LCM.

SOLUTION: We have, $96 = 2^5 \times 3$ and $404 = 2^2 \times 101$

Thus, HCF = $2^2 = 4$

Now, HCF x LCM = Product of numbers

= 96 x 404,

LCM = (96 x 404)/HCF = (96 x 404)/4 = 96 x 101 = 9696

EXAMPLE 3: On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance and complete steps?

SOLUTION: Each person will cover the same distance in complete steps if the distance covered in cm is the LCM of 40, 42 and 45.

Now, $40 = 2^3 \times 5$, $42 = 2 \times 3 \times 7$ and $45 = 3^2 \times 5$

LCM of 40, 42 and 45 is $2^3 \times 3^2 \times 5 \times 7 = 2520$

Hence, minimum distance each should walk = 2520 cm.

EXAMPLE 4: Prove that 5 - √3 is an irrational number.

SOLUTION: Let us assume on the contrary that $5 - \sqrt{3}$ is rational. Then, there exist co-prime positive integers a and b such that

$$5 - \sqrt{3} = a/b$$

$$\Rightarrow 5 - a/b = \sqrt{3}$$

$$\Rightarrow (5b - a)/b = \sqrt{3}$$

Since, a and b are integers and thus $(5b - a)/b$ is rational number. Thus $\sqrt{3}$ is rational but this contradicts the fact that $\sqrt{3}$ is irrational. So, our assumption is incorrect. Hence, $5 - \sqrt{3}$ is an irrational number.

EXAMPLE 5: Prove that 3 + 2√5 is irrational.

SOLUTION: Let us assume on the contrary that $3 + 2\sqrt{5}$ is rational. Then, there exist co-prime positive integers a and b such that

$$3 + 2\sqrt{5} = a/b$$

$$\Rightarrow 2\sqrt{5} = a/b - 3$$

$$\Rightarrow \sqrt{5} = (a - 3b)/2b$$

Since, a and b are integers and thus $(a - 3b)/2b$ is rational number. Thus $\sqrt{5}$ is rational but this contradicts the fact that $\sqrt{5}$ is irrational. So, our assumption is incorrect. Hence, $3 + 2\sqrt{5}$ is an irrational number.

PRACTICE QUESTIONS

MULTIPLE CHOICE (1 mark)

1. If p and q are two distinct prime numbers, then HCF is

   a) 2 b) 0 c) either 1 or 2 d) 1

2. If p and q are two distinct prime numbers, then LCM (p, q) is

   a) 1 b) p c) q d) pq

3. Let p be a prime number. The sum of its factors is

   a) p b) 1 c) p + 1 d) p – 1

4. The LCM of the smallest two digits composite number and the smallest composite number is

   a) 12 b) 20 c) 4 d) 44

5. The HCF of smallest prime number and the smallest composite number is

   a) 2 b) 4 c) 6 d) 8

6. The smallest number divisible by all the natural numbers between 1 and 10 (both inclusive) is

   a) 2020 b) 2520 c) 1010 d) 5040

7. Let n be a natural number. Then, the LCM (n, n+1) is

   a) n b) n + 1 c) n (n + 1) d) 1

8. If 3 is the least prime factor of m and 5 is the least prime factor of n, then the least prime factor of (m + n) is

   a) 11 b) 2 c) 3 d) 5

9. If HCF (x, 8) = 4, LCM (x, 8) = 24, then x is

   a) 8 b) 10 c) 12 d) 14

10. If two positive integers m and n are expressible in the form $m = pq^3$ and $n = p^3q^2$, where p, q are prime numbers, then HCF (m, n) =

    a) pq b) pq^2 c) p^3q^3 d) p^2q^3

11. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and LCM (a, b, c) = $2^3 \times 3^2 \times 5$, then n =

    a) 1 b) 2 c) 3 d) 4

12. If n is any natural number, then $6^n - 5^n$ always end with

    a) 1 b) 3 c) 5 d) 7

ASSERTION AND REASONING (1 mark)

Each of the following examples contains Assertion (A) and Reason (R) has following four choices (a), (b), (c) and (d), only one of which is the correct answer. Mark the correct answer.

(a) A is true, R is true; and R is correct explanation for A.
(b) A is true, R is true; R is not a correct explanation for A.
(c) A is true, R is false.
(d) A is false, R is true.

1. A: If LCM (60, 72) = 360, then HCF (60, 72) = 12.
   R: HCF (a, b) x LCM (a, b) = a + b.
2. A: The product of $(5 + \sqrt{3})$ and $(2 - \sqrt{3})$ is an irrational number.
   R: The product of two irrational numbers is an irrational number.
3. A: HCF and LCM of two natural numbers are 25 and 815 respectively.
   R: LCM of two natural numbers is always divisible by their HCF.
4. A: HCF (234, 47) = 1.
   R: HCF of two co-primes is always 1.
5. A: $\sqrt{11}$ is an irrational number.
   R: If p is a prime number, then $\sqrt{p}$ is an irrational number.

VERY SHORT ANSWER TYPE (2 marks)

1. **Given that HCF (2520, 6600) = 40 and LCM (2520, 6600) = 252 x k, then find the value of 'k'.
2. *If two positive integers a and b are written as $a = p^3q^4$ and $b = p^2q^3$, where p and q are prime numbers, such that HCF (a, b) = p^mq^n and LCM (a, b) = p^rq^s, then find the value of $(m + n)(r + s)$.
3. *Three bells ring at intervals of 4, 7 and 14 minutes. All the three rang at 6 AM. When will they ring together again?
4. *Determine the prime factorization of 58500.
5. **Explain why 7 x 11 x 13 + 13 and 7 x 6 x 5 x 4 x 3 x 2 x 1 are composite numbers.
6. *Check whether 6^n can end with the digit 0 for any natural number n.
7. *The HCF of two numbers is 16 and their product is 3072. Find their LCM.
8. Write the smallest number which is divisible by both 306 and 657.

SHORT ANSWER TYPE (3 marks)

1. **Find the HCF and LCM of 144, 180 and 192 by prime factorization method.
2. Find the largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively.
3. In a school there are two sections – section A and section B of class X. There are 32 students in section A and 36 students in section B. Determine the minimum number of books required for their class library so that they can be distributed equally among students of section A or section B.
4. **Find the LCM and HCF of the pair of integer 404 and 96 and also verify that LCM x HCF = Product of the integers.
5. *On a morning walk, three persons step out together and their steps measure 30 cm, 36 cm and 40 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?
6. *Prove that $\sqrt{2} + \sqrt{5}$ is irrational.
7. *Show that $3 + \sqrt{2}$ is an irrational.
8. If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF, then find the product of two numbers.

LONG ANSWER TYPE (5 marks)

1. Find the largest number which on dividing 1251, 9377 and 15628 leaves remainder 1, 2 and 3 respectively.
2. ***Prove that $\sqrt{2}$ is an irrational number.
3. **Prove that $\sqrt{3}$ is an irrational number.
4. **Prove that $(2 + \sqrt{3})/5$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

CASE STUDY BASED (4 marks)

1. **Mira is very health conscious and avoids fast food, cakes, ice-creams etc. On her birthday she decided to serve fruits to her friend guests. She had 60 bananas and 36 apples which are to be distributed equally among all.**

(i) How many maximum guests Mira can invite? (1 M)

(ii) How many apples will each guest get? (1 M)

(iii) If Mira also decide to distribute 42 mangoes, how many maximum guests she can invite. Also, find the total number of fruits that each guest will get. (2 M)

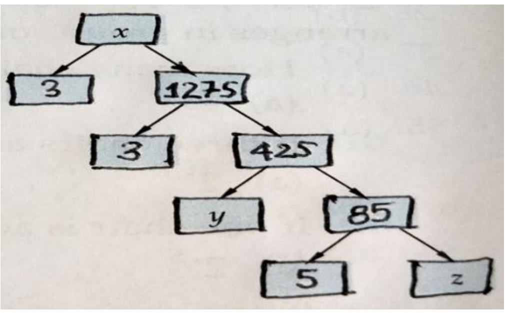
2. ****A seminar is being conducted by an educational organization, where the participants will be educators of different subjects. The number of participants in Hindi, English and Mathematics are 60, 84 and 108 respectively.**

(i) In each room the same number of participants are to be seated and all of them being in the same subject, hence find the maximum number of participants that can be accommodated in each room. (1 M)

(ii) Find the minimum number of rooms required during the event. (1 M)

(iii) Find the product of HCF and LCM of 60, 84 and 108. (2 M)

3. **Observe the factor tree below and answer the questions:



(i) Find the value of y. (1 M)

(ii) Find the value of z. (1 M)

(iii) Determine the value of x + y + z. (2 M)

ANSWER KEY CHAPTER-1

MULTIPLE CHOICE:

1. d 2. d 3. c 4. b 5. a 6. b 7. c 8. b
9. c 10. b 11. b 12. a

ASSERTION AND REASONING:

1. c 2. c 3. d 4. a 5. a

VERY SHORT ANSWER TYPE:

1. 1650 2. 35 3. 6:28 AM 4. $2^2 \times 3^2 \times 5^3 \times 13$

5. Since 7 x 11 x 13 + 13 =13 (7 x 11 + 1) and 7 x 6 x 5 x 4 x 3 x 2 x 1 + 5 = 5(7 x 6 x 4 x 3 x 2 x 1 + 1)

6. $6^n = (2 \times 3)^n = 2^n \times 3^n$ 7. 192 8. 22338

SHORT ANSWER TYPE:

1. HCF = 12 ; LCM = 2880 2. 17 3. 288 Books 4. LCM = 9696 ; HCF = 4 5. 360 cm 8. 194400

LONG ANSWER TYPE:

1. 625

CASE STUDY BASED:

1. (i) 12 (ii) 3 (iii) 6 ; 23
2. (i) 12 (ii) 5 (iii) 45360
3. (i) 5 (ii) 17 (iii) 3847

CHAPTER - 2

POLYNOMIALS

Prepared by- RANJANA KUMARI,
TGT (MATHS)
KV CHAKRADHARPUR

❖ KEY CONCEPTS AND FORMULAE

Polynomial: A polynomial is an algebraic expression in which the exponent on any variable is a whole number. A polynomial is an algebraic expression with variables having positive integral powers only.

General Form:

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + \dots\dots\dots + a_2 x^2 + a_1 x^1 + a_0 x^0$$

Degree of a polynomial: **• The highest power of x in p(x) is called the degree of the polynomial p(x).**

Name of polynomial	degree	Example
Zero	0	0,5,-3….. (constant) $2x^0 = 2$, $6y^0 = 6$
Linear	1	x +1 , y -3, x+z+y etc
Quadratic	2	$6x^2-3y$, $\mathbf{ax^2 + bx + c}$
Cubic	3	$4x^3+5y^2-1$, $x+5y^3-1$
Biquadratic	4	$3x^4+5y^2-1$

❖ Value of a polynomial:

If p(x) is a polynomial in x, and if k is any real number, then the value Obtained by replacing x by k in p(x), is called the value of p(x) at x = k, and is denoted by p(k).

EXAMPLE 1. Find the value of the polynomial $p(x) = 3x^2 + 4x - 4$ where x = 2.

SOLUTION: Given polynomial: p(x) = $\mathbf{3x^2 + 4x - 4}$

Value of given polynomial when x = 2 and we get: $p(2) = 3 \times (2)^2 + 4(2) - 4$

$= 3 \times 4 + 8 - 4 = 12 + 8 - 4 = 16$

Hence the value of $p(x) = 3x^2 + 4x - 4$, where x = 2, is 16.

❖ Zero of a polynomial:

A real number k is said to be a zero of a polynomial p(x), if p(k) = 0

EXAMPLE 2. **What is the value of $p(x) = 2x^3 + 3x + 5$ at $x = -1$**

SOLUTION: We have –

$$P(-1) = 2 \times (-1)^3 + 3 \times (-1) + 5$$

$$= -2 - 3 + 5 = -5 + 5 = 0$$

Here, $p(-1) = 0$

Therefore, –1 is called the zero of cubic polynomial $2x^3 + 3x + 5$.

- ❖ **Linear Polynomial:** A linear polynomial is an expression, in which the degree of the polynomial is 1. The linear polynomial should be in the form of ax+b. Here, "x" is a variable, "a" and "b" are constant.

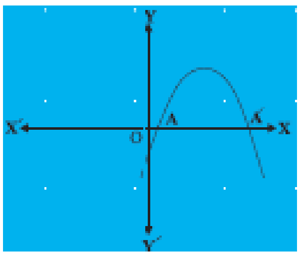
  The polynomial p(x) is ax+b, where $a \neq 0$.

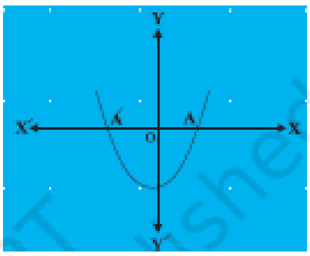
  then the zero of a polynomial is **-b/a = – constant term / coefficient of x**

Quadratic Polynomial:***

A quadratic polynomial is defined as a polynomial with the highest degree of 2. The quadratic polynomial should be in the form of $ax^2 + bx + c$. In this case, $a \neq 0$.

- Standard form of quadratic polynomial **$ax^2 + bx + c$**, where a, b, c are real numbers with $a \neq 0$.
- The zeroes of a polynomial p(x) are precisely the **x-coordinates of the points**, where the graph of y = p(x) intersects the x -axis
- A quadratic polynomial can have at most 2 zeroes
- **The graph of** quadratic polynomial is **Parabola.**





❖ **Zero of a Quadratic polynomial : If α and β** are the zeroes of the quadratic polynomial **Of $ax^2 + bx + c$**

$$\text{sum of zeroes} = \alpha + \beta = -\frac{b}{a} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}, \qquad \alpha + \beta = -\frac{b}{a}$$

$$\text{product of zeroes} = \alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Coefficient of } x^2} \qquad \alpha\beta = \frac{c}{a}$$

Example:

Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

Solution: $x^2 + 7x + 10$

$= x^2 + 2x + 5x + 10$

$= x(x+2) + 5(x+2)$

$= (x + 2)(x + 5)$

Therefore zeroes , $x + 2 = 0$ or $x + 5 = 0$,

i.e., $x = -2$ or $x = -5$.

Therefore, the zeroes of $x^2 + 7x + 10$ are **$\alpha = -2$ and $\beta = -5$**.

Verification:

$$\text{sum of zeroes} = -2 + (-5) = -(7) = \frac{-(7)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2},$$

$$\text{product of zeroes} = (-2) \times (-5) = 10 = \frac{10}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}.$$

Form of Quadratic polynomial when zeroes α and β are given

$$\boxed{x^2 - (\alpha + \beta)x + \alpha\beta}$$

Example1: Find a quadratic polynomial, whose zeroes are – 3 and 2, respectively

Given:

$\alpha = -3 \qquad \beta = 2$

Required quadratic equation is: $x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (-3 + 2)x + (-3)x2$

$= x^2 - (-1)x - 6$

$= x^2 + x - 6$

Example2: Find a quadratic polynomial, ***the sum and product*** of whose zeroes are – 3 and 2, respectively

Solution:
Given: $\alpha + \beta = -3$, $\alpha\beta = 2$
Required quadratic equation is: $x^2 - (\alpha + \beta)\, x + \alpha\beta$
$= x^2 - (-3)\, x + 2$
$= x^2 + 3x + 2$

Question: Write a polynomial whose zeros are $(2+\sqrt{3})$ and $(2 - \sqrt{3})$

EXAMPLE 3. **Find the zeroes of the quadratic polynomial $x^2 - 2x - 8$ and verify the relationship between the zeroes and the coefficients.**

SOLUTION: Here, $p(x) = x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

$\Rightarrow x(x-4) + 2(x-4) = 0$

$\Rightarrow (x-4)(x+2) = 0$

$\Rightarrow x = 4, -2$

Now, $a = 1$, $b = -2$, $c = -8$, $\alpha = 4$, $\beta = -2$

Sum of zeroes, $\alpha+\beta = 4 + (-2) = 2$

$-b/a = -(-2)/1 = 2$

Therefore $\alpha + \beta = -b/a$

Also, product of zeroes $= c/a = -8/1 = -8$

$\alpha\beta = -8 \times 1 = -8$

therefore, $\alpha\beta = c/a$

PRACTICE QUESTIONS

MULTIPLE CHOICE (1 mark)

1. The zeroes of the polynomial x^2+x-2 are:-

a) 1, 2 b) -1, -2 c) 1, -2 d) -1, 2

**2. A quadratic polynomial whose zeroes are 2 and -3/2 is:-

a) $2x^2-4x-3$ b) $2x^2+x-6$ c) $2x^2-x-6$ d) $2x^2-2x-6$

3. The number of polynomials having zeroes as-2 and 5 is:-

a) 1 b) 2 c) 3 d) more than 3

***4. If α and β are the zeroes of the polynomials $2x^2 + 3x +5$, then the value of $1/\alpha + 1/\beta$ is :-

a) -2/3 b) 2/5 c) 3/5 d) -3/5

5. If -3 is one of the zeroes of the quadratic polynomial $(k-1) x^2 + kx -3$, then the value of k is :-

a) -1/2 b) 1/2 c) -2 d) 2

6. The zeroes of the polynomial $3x^2 + 11x -4$ are:-

a) 1/3, -4 b) -1/3 ,4 c) 1/3,4 d) -1/3,-4

**7. If α and β are the zeroes of the polynomial $2x^2 + 7x -3$,then the value of $\alpha^2+\beta^2$ is

a) 49/4 b) 37/4 c) 61/4 d) 61/2

8. If $\propto$, β are the zeros of the polynomial $x^2 - 6x + k$ and $3 \propto + 2\beta = 20$, then value of k is

a) -8 b) 16 c) -16 d) 8

* 9. If 2 and ½ are the zeros of $P(x) = px^2 + 5x + r$, then

a) $p = r =2$ b) $p = r = -2$ c) $p = 2, r = -2$ d) $p = -2, r = 2$

10. The zeros of the quadratic polynomial $x^2 + 99x + 127$ are: -

a) Both Positive b) Both Equal c) Both Negative d) One Positive and One Negative

**11. If $\propto$, β are the zeros of the polynomial $f(x) = x^2 - p(x + 1) - c$ then $(\alpha + 1)(\beta+1)=$

a) $c - 1$ b) $1 - c$ c) c d) $1 + c$

12. What should be added to the polynomial $x^2 - 5x + 4$, so that 3 is a zero of the resulting polynomials?

a) 1 b) 2 c) 4 d) 5

13. **What is the quadratic polynomial whose sum and the product of zeroes is $\sqrt{2}$, ⅓ respectively?**

(a) $3x^2-3\sqrt{2}x+1$ (b) $3x^2+3\sqrt{2}x+1$

(c) $3x^2+3\sqrt{2}x-1$ (d) None of the above

14. **If the zeroes of the quadratic polynomial ax^2+bx+c, $c\neq0$ are equal, then

(a) c and b have opposite signs (b) c and a have opposite signs

(c) c and b have same signs (d) c and a have same signs

15. **Zeroes of $p(x) = x^2-27$ are:**

(a) $\pm9\sqrt{3}$ (b) $\pm3\sqrt{3}$ (c) $\pm7\sqrt{3}$ (d) None of the above

VERY SHORT ANSWER TYPE (2 marks)

1. ***Find the zeroes of the polynomial $9t^2 - 6t + 1$ and verify the relationship between the zeroes and the coefficients. [CBSE,2014]
2. Find the quadratic polynomials whose zeroes are $3 + \sqrt{2}$ and $3-\sqrt{2}$.
3. ** If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q. [CBSE 2012]
4. If 2 and -3 are the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$, then find the values of a and b. [CBSE 2011]
5. **If α, β are the zeroes of the polynomial $2y^2 + 7y + 5$, write the value of $\alpha + \beta + \alpha\beta$. [CBSE 2010]
6. ***Write the polynomial, the product and sum of whose zeroes are -9/2 and -3/2 Respectively [CBSE 2009]
7. If $(x + 1)$ is a factor of $x^2 - 3kx + 3k - 13$, find k.
8. Show that $x^2 + 4x + 7$ has no zero.
9. Check whether $x = -3$ is a zero of $x^3 + 11x^2 + 23x - 35$.
10. Find the zeroes of the polynomial $2x^2 - 9$ and verify the relationship between zeroes and coefficients.

11. ** If α and β are the zeroes of the polynomial $p(x) = x^2 + 5x + k$ such that $\alpha - \beta = 1$.

Find **k.**

12 . Find the value of "x" in the polynomial $2a^2 + 2xa + 5a + 10$ if $(a + x)$ is one of its factors.

13. Write a quadratic equation with roots 3 and 5. [CBSE 2022]

14. Find the quadratic polynomial if its zeroes are 0, $\sqrt{5}$

**15 ****Does the polynomial $a^4 + 4a^2 + 5$ have real zeroes?

SHORT ANSWER TYPE (3 marks)

***1. Find the zeroes of the quadratic polynomial $3x^2$-2 and verify the relationship between the zeroes and the coefficients. [CBSE 2016]

2. If one zero of the quadratic polynomial $f(x) = 4x^2 - 8kx + 8x - 9$ is negative of the other, then the zeroes of $kx^2 + 3kx + 2$. [CBSE 2015]

3. If α and β are zeroes of a polynomial $x^2 + 6x + 9$, then form a polynomial whose zeroes are $-\alpha$ and $-\beta$. [CBSE 2014]

*4. Verify whether 2, 3 and ½ are the zeroes of the polynomial $p(x) = 2x^3 - 11x^2 + 17x - 6$. [CBSE 2013]

5. Show that ½ and -3/2 are the zeroes of the polynomial $4x^2 + 4x - 3$ and verify the relationship between zeroes and coefficients of polynomial. [CBSE 2012]

6. If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are a-b, a, a + b, find the values of a and b.

* 7. If the zeroes of the polynomial $x^2 - 5x + k$ are reciprocals of each other, find the value of k.

8. If the sum of the zeroes of the polynomial $p(x) = (a + 1)x^2 + (2a + 3)x + (3a + 4)$ is -1, then find the product of the zeroes.

9. If (x + a) is a factor of two polynomials $x^2 + px + q$ and $x^2 + mx + n$, then prove that $a = (n - q) / (m - p)$

10. If one zero of a polynomial $3x^2 - 8x + 2k + 1$ is seven times the other, find the value of k.

*11. Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and 1/3 respectively.

12. Find a quadratic polynomial, the sum and product of whose zeroes are 0 and 5 respectively.

13. Obtain the zeros of the quadratic polynomial $3x^2 - 8x + 4$ and verify the relation between its zeros and coefficients.

14. If p and q are the zeroes of the quadratic polynomial $a x^2 + b x + c$ $(a \neq 0)$, find the value of $pq + (p + q)$

**15. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the value of p and q.

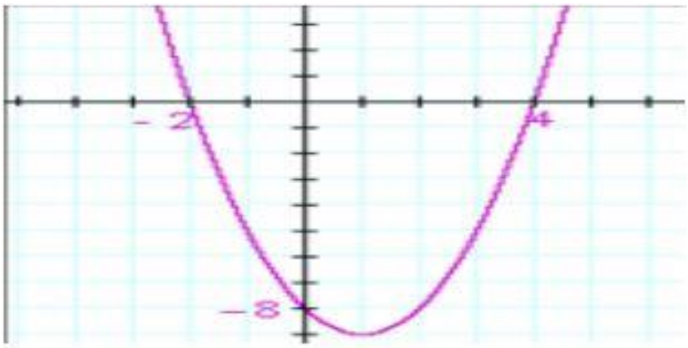
CASE STUDY BASED (4 marks)

Q 1. Asana is a body posture, originally and still a general term for a sitting meditation pose, and later extended in hatha yoga and modern yoga as exercise, to any type of pose or position, adding reclining, standing, inverted, twisting, and balancing poses. In the figure, one can observe that poses can be related to representation of quadratic polynomial.

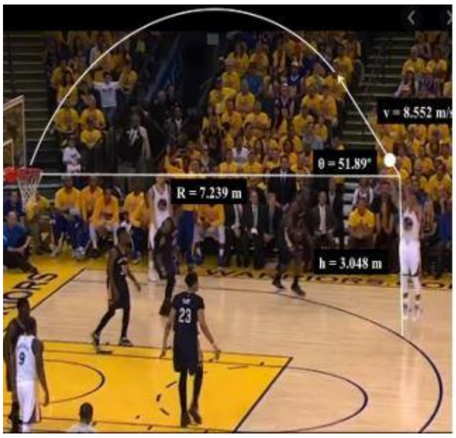
i) What is the shape of the poses shown in above figure? 1M

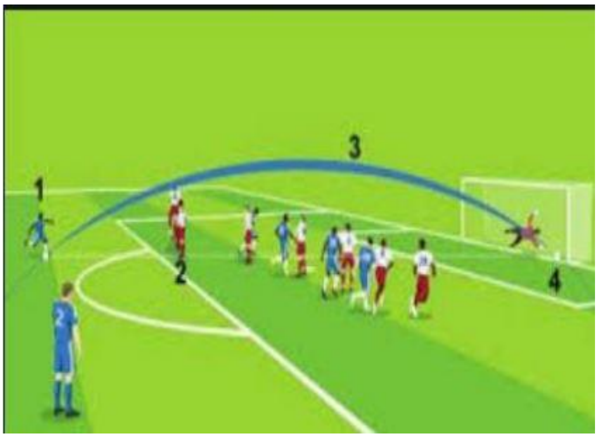
ii) In the graph, how many zeroes are there for the polynomial? 1 M

iii) Find the two zeroes in the below shown graph. 2 M



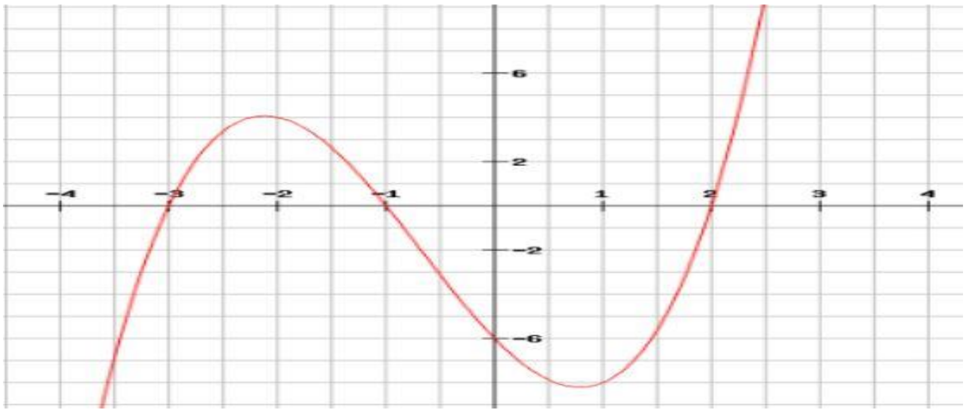
Q2. **Basketball and soccer are played with a spherical ball. Even though an athlete dribbles the ball in both sports, a basketball player uses his hands and a soccer player uses his feet. Usually, soccer is played outdoors on a large field and basketball is played indoor on a court made out of wood. The projectile (path traced) of soccer ball and basketball are in the form of parabola representing quadratic polynomial**





i). What is the shape of the path traced shown in above figure? (1 M)

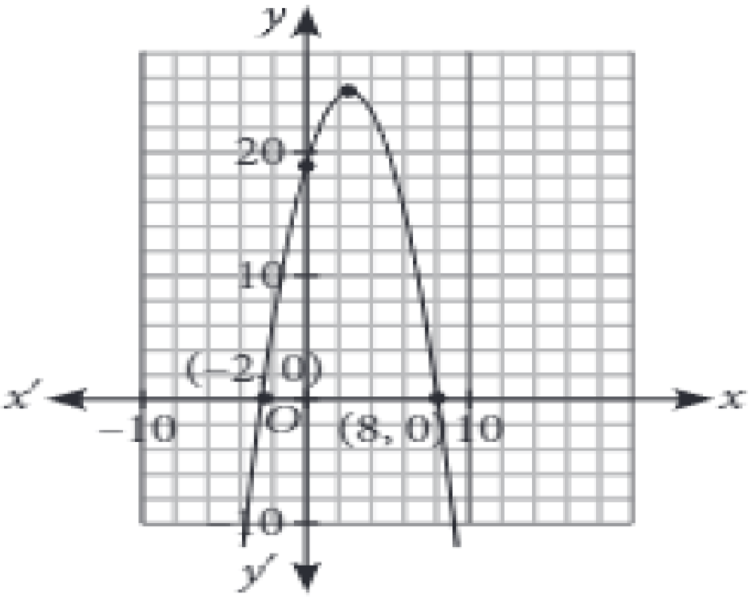
ii). Observe the following graph and answer



In the above graph, how many zeroes are there for the polynomial? (1 M)

iii). What are the three zeroes in the above shown graph? (2 M)

Q3. Priya and her husband Aman who is an architect by profession, visited France. They went to see Mont Blanc Tunnel which is a highway tunnel between France and Italy, under the Mont Blanc Mountain in the Alps, and has a parabolic cross-section. The mathematical representation of the tunnel is shown in the graph.



Based on the above information, answer the following questions.

(i) Find the zeroes of the polynomial whose graph is given above. **(1 M)**

ii) What will be the expression of the polynomial given in diagram? **(1 M)**

iii) If the tunnel is represented by $-x^2+3x-2$. Find its zeroes. **(2 M)**

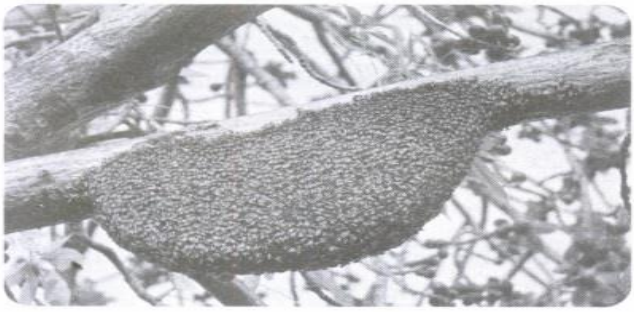
Q4. Pankaj's father gave him some money to buy avocado from the market at the rate of $p(x) = x^2 - 24x + 128$. Let a, β .are the zeroes of p(x). Based on the above information, answer the following questions.

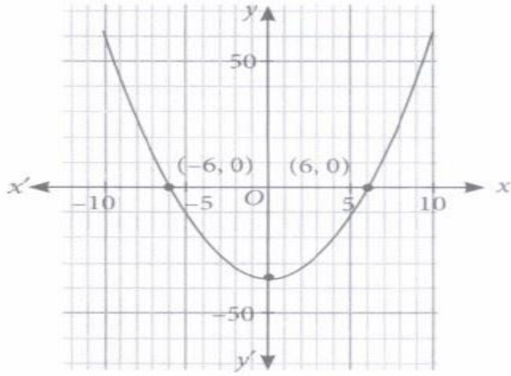
i) Find the value of a and β, where a < β. (1 M)

ii) Find the value of $\alpha + \beta + \alpha\beta$. (1 M)

iii) If α and β are zeroes of $x^2 + x - 2$ then find $(1/\alpha + 1/\beta)$. (2 M)

Q 5. While playing in garden, Sahiba saw a honeycomb and asked her mother what is that. She replied that it's a honeycomb made by honey bees to store honey. Also, she told her that the shape of the honeycomb formed is parabolic. The mathematical representation of the honeycomb structure is shown in the graph.





Based on the above information, answer the following questions.

i) What is the shape of graph of a quadratic polynomial? (1 M)

ii) Find the sum of zeroes of the polynomial $x^2 + 2x – 3$. (1 M)

iii) What is the expression of the polynomial represented by the graph ? (2 M)

LONG ANSWER TYPE (5 marks)

***Q1 . If one zero of the quadratic polynomial $f(x) = 4x^2 – 8kx + 8x – 9$ is negative of the other , then find the zeroes of $kx^2 + 3kx + 2$. [CBSE 2015]

*Q 2. If p and q are the zeroes of the polynomial $t^2 - 4t + 3$, show that $1/p + 1/q - 2pq + 14/3 = 0$

Q3. If α , β and γ are zeroes of the polynomial $6x^3 + 3x^2 - 5x + 1$, then find the value of $\alpha^{-1} + \beta^{-1} + \gamma^{-1}$

**Q4 .If α and β are zeroes of the polynomial $2x^2 - 5x + 7$,then find a quadratic polynomial whose zeroes are $3\alpha + 4\beta$ and $4\alpha + 3\beta$.

**Q 5. If α and β are the zeroes of the quadratic polynomial $x^2 - x - 6$, then find the values of :

a) $\alpha^3 + \beta^3$ b) $\alpha^4 + \beta^4$ c) $\alpha - \beta$

ANSWER KEY

MULTIPLE CHOICE:

Q 1. c)	Q 2. c)	Q3.d)	Q4.d)	Q5. d)
Q6. a)	Q7. (c)	Q8.c)	Q9.b)	Q10.b)
Q11. b)	Q12.b)	Q13.a)	Q14.d)	Q15.b)

VERY SHORT ANSWER TYPE:

Q1 .sum of zeroes =2/3
Product of zeroes=1/9
Q2. Required polynomial is $x^2 – 6x + 7$
Q3. P= -5 and q = - 6
Q4. a = 0 and b= - 6
Q5. -1
Q6. Required quadratic polynomial is $2x^2 +3x – 9$
Q7. k = 2
Q9. X = - 3 is not a zero .
Q10. $3/\sqrt{2}$ and $-3/\sqrt{2}$
Q11. k = 6
Q12. x = 2
Q13. $x^2 – 8x +15$.
Q14. $x^2 - \sqrt{5}\, x$
Q15. The given polynomial does not have real roots

SHORT ANSWER TYPE:

Q1. $x = +\sqrt{2/3}, -\sqrt{2/3}$
Sum of zeroes =0
Product of zeroes = -2/3
Q2. Zeroes of p(x) are -1 and -2.
Q3. Required quadratic polynomial is $x^2 - 6x+9$.
Q4. p(2), p(3), p(1/2) are the zeroes of p(x) .
Q5. Sum of zeroes = -1
Product of zeroes = -3/4
Q 6. Here a = 1 and b = $\sqrt{2}, -\sqrt{2}$.
Q7. k = 1
Q 8. k = - 2
Q9. Proof must be correct.
Q 10. k =2/3
Q11. $3x^2- 3\sqrt{2}x +1$
Q 12. $x^2+ \sqrt{5}$
Q13. Zeroes are 2 and 2/3
Q14. p q + (p+ q) = (c-b)/a
Q 15. p = -5 and q = -6

CASE STUDY BASED:

Q1 i) parabola
ii) 2
iii) – 2, 4

Q2 i) parabola
ii) 3
iii) – 3, – 1, 2

Q3 i) – 2, 8
ii) $-x^2 + 6x + 16$
iii) 1, 2

Q4 i) 8, 16
ii) 152
iii) 1/2

Q5 i) parabolic
ii) - 2
iii) $x^2 - 36$

LONG ANSWER TYPE:

Q 1. Zeroes of p(x) are -1 and -2.

Q2 .proof must be correct.

Q3. $\alpha^{-1} + \beta^{-1} + \gamma^{-1} = 5$.

Q4.Required quadratic polynomial is $2x^2 - 35x + 157$.

Q5 a) 19 b) 97 c) +5 and _5

******

CHAPTER - 3

LINEAR EQUATIONS IN TWO VARIABLES

Prepared by- ABHIRAM BANRA,
TGT (MATHS)
KV MEGHAHATUBURU

Basic Concepts:

- **Equation:** An algebraic expression with 'equal to'(=) sign is called the equation. It is an algebraic expression of equality.

  Example: $4x=5$, $3y-4=5$, $3x+4y=5$ etc.
- **Linear Equation:** If the greatest exponent of the variables in an equation is one, the equation is said to be a linear equation.

  Example: $3x-4=2$, $4x+6y=7$ etc.
- **Linear Equation in One Variable:** An equation of the form $ax+b=0$, where a, b are real numbers and $a\neq0$ is called linear equation in one variable.

  Example: $2x+7=0$.
- **Linear Equation in Two Variables':** An equation of the form $ax+by+c=0$, where a, b and c are real numbers where a, $b\neq0$ is called linear equation in two variables.

  Example: $9x-2y+3=0$
- **Solution of an Equation:** That value/values of variable/variables used in equation, which make(s) two sides of equation equal *or* satisfy the equation is called solution of the equation.

  Example: For equation $3x-4=2$, $x=2$ is a solution.

  For equation $5x-2y=4$, $(x,y)=(2,3)$ is one of solutions.
- **Pair of Linear Equations in Two Variables:** Two linear equations of the form $ax+by+c=0$ taken together form a pair of linear equations in two variables.

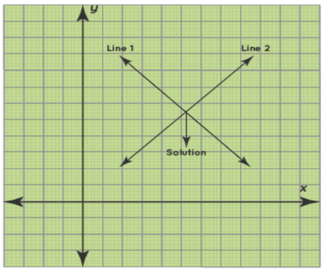
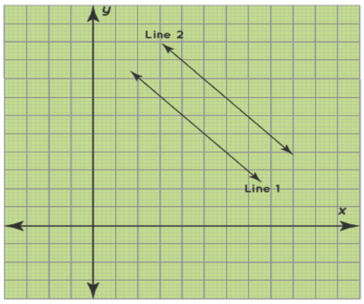
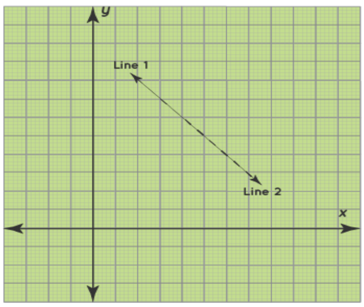
  Example: $2x-3y+4=0$ and $3x+5y-7=0$.
- **Solutions of Linear equation in Two Variables:** The values of x and y satisfying each one of the given pair of linear equation is called their solutions.

  Example: The solution of pair of linear equations

  $2x+3y-8=0$ and $5x-2y-9=0$ is $x=1,y=2$

- Solutions of Linear equation in Two Variables
  - Graphical Method
    - Single Solutions (If lines intersect at single Point)
    - Infinitely many Solutions(If lines Coincide)
    - No Solution(If lines are Parallel)
  - Algebraic Method
    - Substitution Method
    - Elimination Method

Types of Solutions and their Graphs

Type of solution	Conditions	Graphical Representation
Unique Solution (Consistent and Independent)	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	 Unique Solution (Consistent and independent)
No Solution (Inconsistent and Independent)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	 No Solution (Inconsistent and independent)
Infinite Number of Solutions (Consistent and Dependent)	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	 Infinite number of solutions (consistent and dependent)

- **Interpretation of the pair of linear Equations:**

The general form of a pair of linear equations is

$$a_1x+b_1y+c_1=0$$
$$a_2x+b_2y+c_2=0$$

where $a_1 \neq 0$, $b_1 \neq 0$, $a_2 \neq 0$, $b_2 \neq 0$

Solved Questions:

1. **Solve : the following pair of linear equations:**

$3x - 5y = 4$,

$2y + 7 = 9x$.

Sol- Given, $3x - 5y = 4$.....(1),

$9x - 2y = 7$ .....(2)

Multipy eq (1) by 3 and eq (2) by 1 and subtracting eq (2) from (1) we get

$(3x - 5y)3 - (9x - 2y)1 = 4 \times 3 - 7 \times 1$

i.e. $-13y = 5$ $\therefore y = \frac{-5}{13}$

Then put $y = \frac{-5}{13}$ in eq. (1) then We get $x = \frac{9}{13}$

Ans : $x = \frac{9}{13}$, $y = \frac{-5}{13}$

2. **Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?**

Sol- Let the present age of Jacob and his son be x and y respectively.
According to the question,

$(x + 5) = 3(y + 5)$

Or, $x - 3y = 10$ ……………………………………….. (1)

Or, $(x - 5) = 7(y - 5)$

Or, $x - 7y = -30$ ………………………………………. (2)

From (1), we get $x = 3y + 10$ ……………………. (3)

Substituting the value of x in (2), we get

$3y + 10 - 7y = -30$

Or, $-4y = -40$

$\therefore y = 10$ ………………… (4)

Substituting the value of y in (3), we get

$x = 3 \times 10 + 10 = 40$

Hence, the present age of Jacob's and his son is 40 years and 10 years respectively.

3. Raman's age is three times he sum of the ages of his two sons. After 5 years his age will be twice the sum of the ages of his two sons. Find the age of Raman.

Sol- Let x be the present age of Raman's and y be the present ages of sum of his two sons.
ATQ,

$x = 3y$ -----(1)

After 5 years :

Raman's age = $x + 5$

The sum of the ages of his two sons = $y + 5 + 5$
$= y + 10$

ATQ,

$x + 5 = 2(y + 10)$

Or ,$x + 5 = 2y + 20$

Or ,$x - 2y = 20 - 5$

Or, $x - 2y = 15$ --------(2)

By applying the value of x in (2), we get

$3y - 2y = 15$ {from eqn 1}

$\therefore\ y = 15$

Put the value of y in eqn (1) $x = 3(15)$

$\therefore\ x = 45$

Answer : So, Raman present age is 45 years old.

4. The middle digit of a number between 100 and 1000 is zero and the sum of the other digit is 13. If the digits are reversed, the number so formed exceeds the original number by 495. Find the number.

Sol-The required number will be in the form X 0 Y
Middle digit = 0

$x + y = 13$ -------(1)

$Y0X = X0Y + 495$

$100y + x = 100x + 1y + 495$

$x - 100x + 100y - y = 495$

$-99x + 99y = 495$

$-x + y = 5$ -------(2)

$x = y - 5$

By applying the value of x in (1), we get

$y - 5 + y = 13$

$2y = 13+5$

$2y = 18$

$y = 9$

When $y = 9$,

$x = 9 - 5$

$x = 4$

So, the required number is 409.

5. **Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$**.

Sol- $2x + 3y = 11$…………………………..(I)

$2x - 4y = -24$………………………… (II)

From equation (II), we get

$x = (11-3y)/2$ ………………….(III)

Substituting the value of x in equation (II), we get

$2(11-3y)/2 – 4y = 24$

$11 – 3y – 4y = -24$

$-7y = -35$

$y = 5$……………………………………..(IV)

Putting the value of y in equation (III), we get

$x = (11-3×5)/2 = -4/2 = -2$

Hence, $x = -2$, $y = 5$

Also,

$y = mx + 3$

$5 = -2m +3$

$-2m = 2$

$m = -1$

Therefore the value of m is -1.

6. **The sum of numerator and denominator of a fraction is 3 less than twice the denominator. If each of the numerator and denominator is decreased by 1, the fraction becomes ½. Find the fraction.**

Solution: Let the numerator be x and denominator be y.

Let the fraction be x/y

ATQ,

$x+y+3=2y$

$⇒x−y=−3$

$x=y−3$ ………(i)

Also,

$(x-1)/(y-1) = ½$

$⇒ (x-1) = (y-1)/2$

$⇒2x=y−1+2=y+1$

$2x−y=1$ …………..(ii)

Substituting (i) in equation (ii), we get;

$2(y−3)−y=1$

$⇒2y−6−y=1$

$⇒y=7$

Now, put y=7 in (i) we get;

$x=7−3=4$

Thus, x=4,y=7

Therefore, the required fraction is 4/7.

7. **If the system of equations has a unique solution, find the value of k.**

6x + 2y = 3 and kx + y = 2

Solution: Given,

$6x + 2y = 3$ and $kx + y = 2$ have unique solutions

Thus, $a1/a2 \neq b1/b2$

$6/k \neq 2/1$ $\qquad k \neq 3$

Therefore, k will have any real value apart from 3.

Q.8 Read the following text and answer the following questions on the basis of the same: Places A and B are 100 km apart on a highway . One car starts from A and another from B at the same time .If the cars travel in the same direction at different speeds , they meet in 5 hours . If they travel towards each other ,they meet in 1 hour.

a) What is the actual speed of the other car ?

i) 40 km/hr ii) 60km/hr iii) 20km/hr iv))100km/hr

b) What is the relative speed of both cars while they are travelling towards each other?

i) u+v km/hr ii)u-v km/hr iii)u/v km/hr iv)uv km/hr

c) The given problem is based on which mathematical concept

i)polynomial ii)pair of linear equations iii)quadratic iv) none of these

Answer		
a	b	c
ans (i)	ans (i)	ans (ii)

SECTION-A (MCQ - 1 Marks)

1*. Graphically, the pair of equation

$6x-3y+10=0$

$2x-y+9=0$

Represents two lines which are

(A) Intersecting at exactly one point (B) Intersecting at exactly two point

(C) Coincident (D) Parallel

2*. The pair of equation $x+2y+5=0$ and $-3x-6y+1=0$ have :

(A) A unique solution (B) Exactly two solutions

(C) Infinitely many solutions (D) No Solution

3**. If a pair of linear equations is consistent, then the lines will be:

(A) Parallel (B) Always coincident

(C) Intersecting or coincident (D) Always interesting

4**. The pair of equation x=a and y=b graphically represents lines which are:
(A) Parallel (B) Intersecting at (b, a)
(C) Coincident (D) Intersecting at (a, b)

5**. The pair of equation y = 0 and y = -7 has:
(A) One solution (B)Two solutions
(C) Infinitely many solutions (D) No solution

6*. One equation of a pair of dependent linear equations is -5x+7y=2. The second equation can be:
(A) 10x+14y+4=0 (B) -10x-14y+4=0
(C) -10x+14y+4=0 (D) 10x-14y = -4

7***. For what value of k, do the equations 3x-y+8=0 and 6x-ky=-16 represents coincident lines?
(A) 12 (B) −12 (C) 2 (D) -2

8***. If the lines given by 3x+2ky=2 and 2x+5y+1=0 are parallel, then the value of k is
(A) −5/4 (B) 2/5 (C) 15/4 (D) 3/2

9**. A pair of linear equation which has a unique solution x=2, y=-3 is
(A) x+y=-1 and 2x-3y=-5 (B) 2x+5y=-11 and 4x+10y=-22
(C) 2x-y=1 and 3x+2y=0 (D) x-4y-14=0 and x-y-13=0

10***. Shweta has only ₹1 and ₹2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is ₹75, then the number of ₹1 and ₹2 coins are, respectively
(A) 35 and 15 (B) 35 and 20 (C) 15 and 35 (D) 25 and 25

11**. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively:
(A) 4 and 24 (B) 5 and 30 (C) 6 and 36 (D) 3 and 24

12**. If x=a, y=b is the solution of the equations x-y=2 and x+y=4, then the values of a and b are, respectively
(A) 3 and 5 (B) 5 and 3 (C) 3 and 1 (D) -1 and 3

13*. The larger of the two supplementary angles exceed the smaller by 18°, then the angles are:
(A) 99°, 81° (B) 98°, 82° (C) 97°, 83° (D) None of these
.

14**. x and y are 2 different digits. If the sum of the two digit numbers formed by using both the digits is a perfect square, then value of x +y is
(a) 10 (b) 11 (c) 12 (d) 13

15***. In a number of two digits, unit's digit is twice the tens digit. If 36 be added to the number, the digits are reversed. The number is
(a) 36 (b) 63 (c) 48 (d) 84

16**.The value of k for which the system of equation x+y-4=0 and 2x + ky =3 has no solution is
(a) -2 (b) ≠2 (c) 3 (d) 2

SECTION-B (2 Marks)

1*. Given the linear equation $3x+4y = 9$. Write another linear equation in these two variables such that the geometrical representation of the pair so formed is:

(1) intersecting lines (2) coincident lines.

2***. For what value of 'p' does the pair of linear equations given below has unique solution?

$4x+ py +8 = 0$ and $2x+2y + 2 =0$.

3**. Is the system of linear equations $2x+3y - 9 =0$ and $4x+6y - 18 =0$ consistent? Justify your answer.

4*. Two lines are given to be parallel. The equation of one of the lines is $4x + 3y = 14$, then find the equation of the second line.

5***. Find the value(s) of k for which the pair of linear equations $kx + y = k2$ and $x + ky = 1$ have infinitely many solutions.

6***. Find the value of 'c' for which the pair of equations $cx-y =2$ and $6x-2y =3$ will have infinitely many solutions.

7**. If the lines given by $4x+5ky=10$ and $3x+y+1=0$ are parallel, then find value of 'k'.

8*. Do the equations $4x+3y-1=5$ and $12x + 9y =15$ represent a pair of coincident lines?

9**. For what value of 'k', does the systems of linear equations $2x + 3y = 7$,

$(k-1)x + (k + 2)y = 3k$ have an infinite number of solutions?

10*. Solve the pair of linear equations.

$3x + 4y = 10$ and $2x – 2y = 2$.

SECTION-C (3 marks)

1**. Solve graphically: $2x - 3y + 13 = 0$; $3x - 2y + 12 = 0$

2**. Find the value of k for which the following pair of equations has no solution :

$x + 2y = 3$, $(k-1)x+(k+1)y=(k+2)$

3*. Solve $x + y = 5$ and $2x - 3y = 4$ by elimination method and the substitution method.

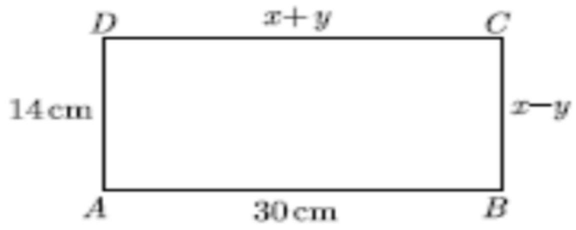
4***. Draw the graph of the following equations:

$2x - y = 1$, $x + 2y = 13$,

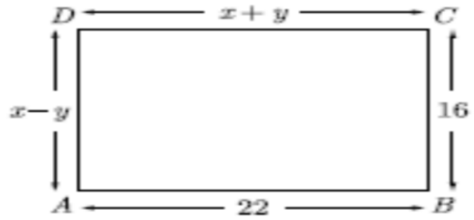
Find the solution of the equations from the graph and shade the triangular region formed by the lines and the y -axis.

5**. Solve the following pair of linear equations graphically: $x - y = 1$, $2x + y = 8$. Also find the co-ordinates of the points where the lines represented by the above equation intersect y - axis.

6*. In Figure, ABCD is a rectangle. Find the values of x and y



7*. In Figure, ABCD is a rectangle. Find the values of x and y



8**. Half the perimeter of a rectangular garden, whose length is 4 m more then its width, is 36 m. Find the dimensions of garden.

9***. Determine graphically whether the following pair of linear equations :

$3x - y = 7$; $2x + 5y + 1 = 0$ has : unique solution infinitely many solutions or no solution.

10**. Solve : $99x + 101y = 499$, $101x + 99y = 501$

SECTION-D (5 Marks)

1*. The ratio of incomes of two persons is 11:7 and the ratio of their expenditures is 9:5. If each of them manages to save Rs 400 per month, find their monthly incomes.

2***. A fraction becomes 9/11 if 2 is added to both numerator and denominator. If 3 is added to both numerator and denominator it becomes 5/6. Find the fraction.

3***. 2 man and 7 boys can do a piece of work in 4 days. It is done by 4 men and 4 boys in 3 days. How long would it take for one man or one boy to do it?

4**. Solve the following pair of equations graphically: $2x + 3y = 12$, $x - y - 1 = 0$. Shade the region between the two lines represented by the above equations and the X -axis.

5**. A chemist has one solution which is 50% acid and a second which is 25% acid. How much of each should be mixed to make 10 litre of 40% acid solution.

6**. If $2x+y = 23$ and $4x-y=19$, find the value of $(5y-2x)$ and $(\frac{y}{x} - 2)$.

7***. A train covered a certain distance at a uniform speed. If the train would have been 6 km/h faster, it would have taken 4 hours less than the scheduled time and if the train were slower by 6km/h, it would have taken 6 hours more than the scheduled time. Find the length of the journey.

8***. A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

9*. Students of a class x are made to stand in rows. If one student is extra in each row, there would be 2 rows less. If one student is less in each row, there would be 3 rows more. Find the number of students in the class x.

10*. Draw the graph of pair of linear equations :

$x + y = 5, \qquad 2x + 2y = 10$

SECTION-E (Case Study Questions – 4 marks)

1. * It is common that governments revise travel fare from time to time based on various factors such as inflation (a general increase in prices and fall in purchasing value of money) on different types of vehicles like auto, rickshaws, taxis, radio cab etc. The auto charges in a city comprise of a fixed charge together with the charge for the distance covered. Study the following situations:

Name of the city	Distance travelled (km)	Amount Paid (Rs)
City A	10	75
	15	110
City B	8	91
	14	145

Situation1: In city A, for a journey of 10 km, the charge paid is Rs 75 and for a journey of 15 km, the charge paid is Rs 110.

Situation2: In a city B, for a journey of 8 km, the charge paid is Rs 91 and for a journey of 14 km, the charge paid is Rs 145.

i) If the fixed charges of auto rickshaw be Rs *x* and the running charges be Rs *y* km/h , the pair of linear equations representing the situation is

a) $x + 10y = 110, x + 15y = 75$
b) $x + 10y = 75, x + 15y = 110$
c) $10x + y = 110, 15x + y = 75$
d) $10x + y = 75, 15x + y = 110$

ii) A person travels a distance of 50km. The amount he has to pay is

a) Rs 155 b) Rs 255 c) Rs 355 d) Rs 455

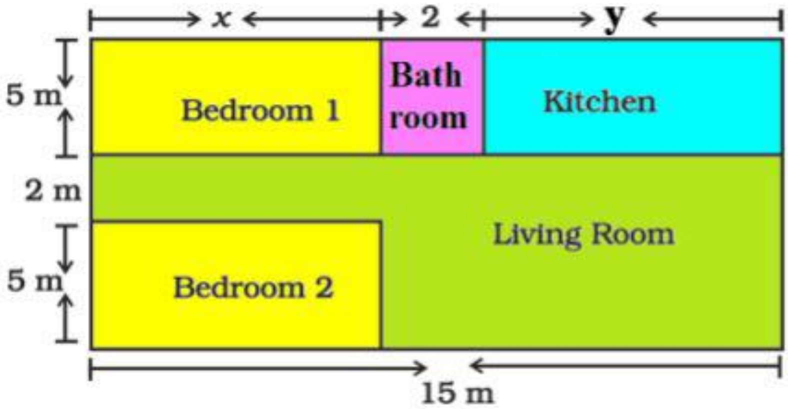
iii)What will a person have to pay for travelling a distance of 30 km?

a) Rs 185 b) Rs 289 c) Rs 275 d) Rs 305

iv)Out of both the city, which one has cheaper fare?

a) City A b) City B c) Both are same d) cannot be decided

2. * Amit is planning to buy a house and the layout is given below. The design and the measurement has been made such that areas of two bedrooms and kitchen together is 95m^2.**



i). Form the pair of linear equations in two variables from this situation.

ii). Find the length of the outer boundary of the layout.

iii). Find the area of each bedroom and kitchen in the layout.

iv). Find the area of living room in the layout.

v). Find the cost of laying tiles in kitchen at the rate of Rs. 50 per sq.m.

3. ** A test consists of 'True' or 'False' questions. One mark is awarded for every correct answer while ¼ mark is deducted for every wrong answer. A student knew answers to some of the questions. Rest of the questions he attempted by guessing. He answered 120 questions and got 90 marks.

Type of Question	Marks given for correct Answers	Marks deducted for wrong Answers
True/ False	1	0.25

i) If answer to all questions he attempted by guessing were wrong, then how many questions did he answer correctly?

ii) How many questions did he guess?

iii) If answer to all questions he attempted by guessing were wrong and answered 80 correctly, then how many marks he got?

iv) If answer to all questions he attempted by guessing were wrong, then how many questions answered correctly to score 95 marks?

ANSWER KEY

CHAPTER-3 PAIR OF LINEAR EQUATION IN TWO VARIABLES

Answers Section A

1. d	2. d	3. c	4.d	5. d	6. d	7.c	8.c	9.b,d	10.d	11.c	12.c	13.a	14.b	15.c	16.d

Answers (Section B)

1. One of the possible equation 3x-5y=10
One of the possible equation 6x+8y=18
2. p≠4
3. Consistent
4. One of the possible solution 12x+9y=5
5. K=1 or k= -1
6. There is no value of 'c' for which lines have infinitely many solutions.
7. k = $\frac{4}{15}$
8. Given pair of lines do not represent a pair of coincident lines.
9. k = 7
10. x=2, y=1.

Answers (Section C)

1. x= -2 and y=3 2.k=3 3.x=19/5 and y=6/5 4.Draw the graph 5.Draw the graph
6.x=22 and y=8 7.x=19 and y=3 8.Length =20m and width =16m 9.Unique solution 10.x=2 , y=3

Answer (Section D)

1. 2200 and1400 2. 7/9 3. 15 days 4. Draw the Graph 5. x=6 ,y=4 6. x=7, y=9 , 31, $\frac{-5}{7}$ 7. 720 km 8. Speed of the stream is 3 km/h and speed of boat is 8km/h
9. 60 Students

Answer (Section E)

1. Answer (Case Study QN-1)

i) b ii) c iii) b iv) a

2. Answer (Case Study QN-2)

i) 2x + y =19, x + y = 13
ii) Length of outer boundary = 12 + 15 + 12 + 15 = 54m
iii) area of bedroom = 5 x 6 = 30m^2, area of kitchen = 5 x 7 = 35m^2
iv) Area of living room = (15 x 7) – 30 = 105 – 30 = 75 sq.m
v) Total cost of laying tiles in the kitchen = Rs50 x 35 = Rs1750

3. Answer (Case Study QN-3)

i) 96 ii) 24 iii) 70 iv) 100

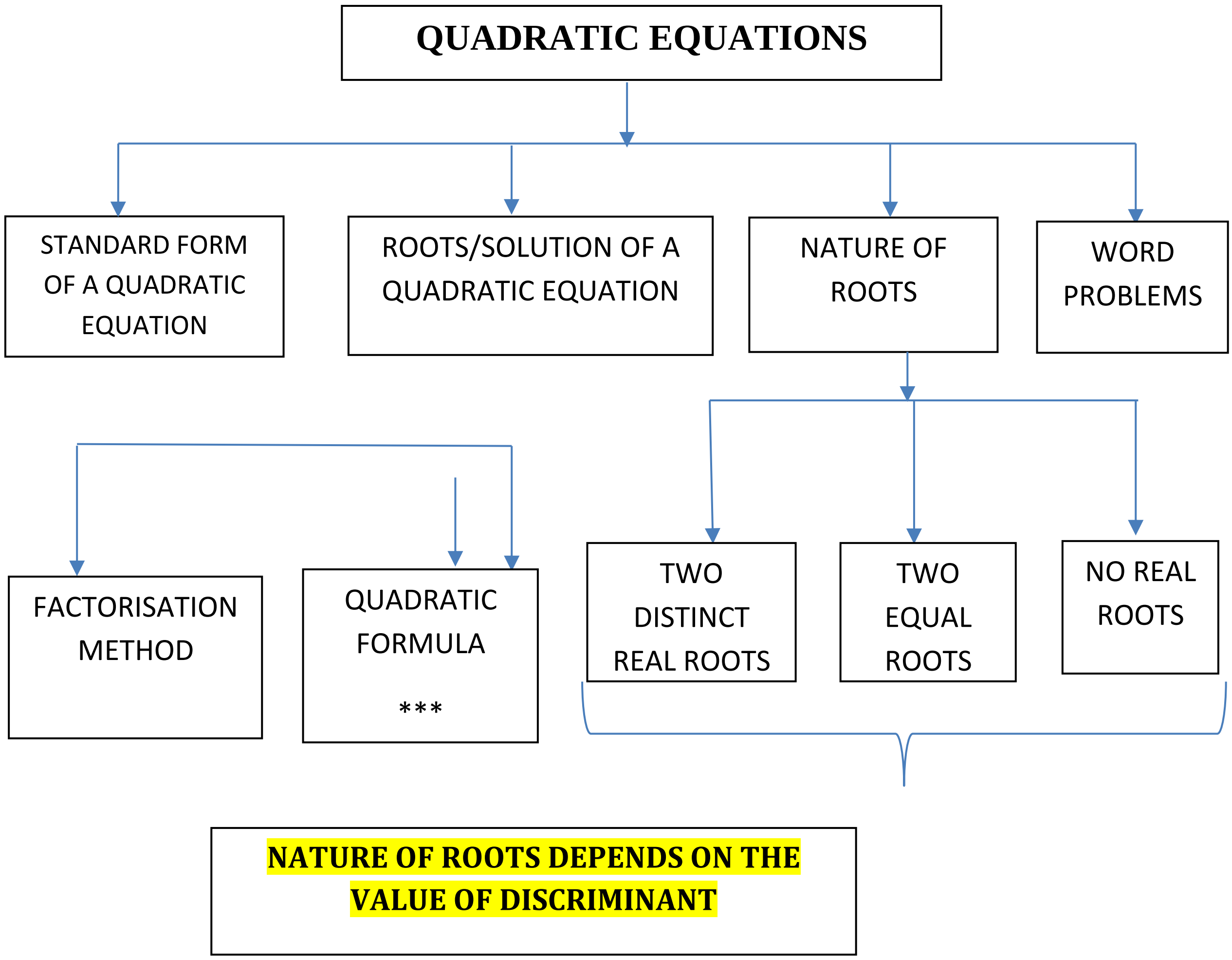
******

CHAPTER -4

QUADRATIC EQUATIONS

Prepared by- ROOPSHREE
TGT (MATHS)
KV CRPF RANCHI

MIND MAP



IMPORTANT NOTES

1. Standard form of a quadratic equation is $\boldsymbol{ax^2} + \boldsymbol{bx} + \boldsymbol{c} = \mathbf{0}$ *where* $a \neq 0$, where a, b & c are real numbers. It has **at most two roots** generally called as α *and* β.

2. A Quadratic equation can be solved by-

- Factorisation method or
- Quadratic formula

Quadratic formula is

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

3. $\boldsymbol{b}^2 - \boldsymbol{4ac}$ is called DISCRIMINANT.

4. A quadratic equation has-

- two distinct real roots if $\boldsymbol{b^2 - 4ac} > 0$
- two equal real roots if $\boldsymbol{b^2 - 4ac} = 0$
- no real roots if $\boldsymbol{b}^2 - \boldsymbol{4ac} < 0$

5. The degree of a quadratic equation is 2.

6. **The sum of the roots of a quadratic equation = Coefficient of x / coefficient of** $\boldsymbol{x^2 = -\frac{b}{a}}$

The product of the roots of a quadratic equation = constant term/coefficient of $\boldsymbol{x^2 = \frac{c}{a}}$

SOLVED EXAMPLES

STANDARD FORM OF A QUADRATIC EQUATION

Example 1. Write $2x^2 = 3x - 7$ in the standard form.

Solution: standard form of $2x^2 = 3x - 7$ is

$2x^2 - 3x + 7 = 0$, where a = 2, b = -3 and c = 7

Example 2. Check whether the following equation is a quadratic equation:

a). $4x^2 = 2(x^2 - x + 1)$

b). $4(x - 3) = -(x^2 - 6x)$

Solution: $4x^2 = 2(x^2 - x + 1)$

Or, $4x^2 = 2x^2 - 2x + 2$

Or, $4x^2 - 2x^2 + 2x + 2 = 0$

Or, $2x^2 + 2x + 2 = 0$

Or, $x^2 + x + 1 = 0$

It is of the form $\boldsymbol{ax^2 + bx + c = 0}$. So, the given equation is a quadratic equation.

SOLUTION OF A QUADRATIC EQUATION

Example 3. Solve the quadratic equation $x^2 + 7x + 10 = 0$ by factorisation.

OR

Find the roots of the quadratic equation $x^2 + 7x + 10 = 0$ by factorisation.

Solution:

$x^2 + 7x + 10=0$

or, $x^2 + 2x + 5x + 10 = 0$

or, $x (x + 2) + 5(x + 2) = 0$

or, $(x + 5) (x + 2) = 0$

Thus, $(x + 2)$ and $(x + 5)$ are the factors of the given quadratic equation.

Solving these two linear factors, we get x = -2, -5 as roots.

Example 4. Find the roots of the quadratic equation $x^2 + 4x – 21 = 0$ using quadratic formula.

Solution:

Given, $x^2 + 4x – 21 = 0$

Here, a = 1, b = 4, c = -21

$b^2 – 4ac = (4)^2 – 4x (1) x (-21) = 16 + 84 = 100$

Substituting these values in the quadratic formula, we get;

Or, $x = \frac{[-4 \pm \sqrt{100}]}{2(1)}$

Or, $x = \frac{(-4 \pm 10)}{2}$

Or, $x = \frac{(-4 + 10)}{2}$, $x = \frac{(-4 - 10)}{2}$

Or, $x = \frac{6}{2}$, $x = \frac{-14}{2}$

Or, x = 3, x = -7

Therefore, the roots of the given quadratic equation are 3 and -7

DISCRIMINANT AND NATURE OF ROOTS OF A QUADRATIC EQUATION

Example 5. Find the value of 'p', if the following quadratic equation has equal roots:
$4x^2 - (p - 2)x + 1 = 0$

Solution:

We have the quadratic equation, **$4x^2 - (p - 2)x + 1 = 0$**

Here a= 4, b= - (p-2) and c= 1

For equal roots, discriminant = 0

Or, $b^2-4ac=0$

Or, $(p-2)^2 - 4 \times 4 \times 1 = 0$

Or, $p^2-4p+4-16=0$

Or, $p^2-4p-12=0$

Or, $p^2-6p+2p-12=0$

Or, $p(p-6)+2(p-6)=0$

Or, $(p-6)(p+2)=0$

Or, **$p=6$ or $p=-2$**

PRACTICE QUESTIONS:

VERY SHORT ANSWER TYPE & MULTIPLE CHOICE QUESTIONS (1 MARK)

Q1. What is the positive root of $\sqrt{3x^2+6} = 9$ *

(a) 3 (b) 5 (c) 0 (d) None of these

Q2. For what value(s) of α quadratic equation $3\alpha x^2 - 6x + 1 = 0$ *has no real roots*? **

(a) $\alpha > 3$ (b) $\alpha < 3$ (c) $\alpha = 3$ (d) None

Q3. Find the nature of the roots of the Quadratic equation $2x^2 - 4x + 3 = 0$? **

(a) Real roots (b) No real roots (c) Equal roots (d) None

Q4. Find the positive values of k for which the Quadratic equation $x^2 + kx + 64 = 0$ and $x^2 - 8x + k = 0$, both will have the real roots? ***

(a) k = 4 (b) k =16 (c) k > 16 (d) k< 4

Q5. If the sum of the roots of the quadratic equation $3x^2 + (2k + 1) - (k + 5) = 0$ is equal to the product of roots, then the value of k is **

(a) 2 (b) 3 (c) 4 (d) 5

Q6. If the equation $x^2 - bx + 1 = 0$ does not possess real roots, then **

(a) $-3 < b \leq +3$ (b) $-2 < b \leq +2$ (c) b > 2 (d) $b < -2$

Q7. Find the roots of the quadratic equation $x - \frac{1}{x} = 0$ is **

(a) $\frac{3}{2}, -\frac{3}{2}$ (b) $\frac{(3+\sqrt{13})}{2}, \frac{(3-\sqrt{13})}{2}$ (c) $\frac{2}{3}, -\frac{2}{3}$ (d) None

Q8. If $(x + 4)(x - 4) = 9$, then the values of x are *

(a) ± 5 (b) ± 1 5 (c) 5, 5 (d) 1 5, 1 5

Q9. How many real roots does the equation $(x + 1)^2 - x^2 = 0$ have? *

(a) 1 (b) 2 (c) 3 (d) 4

Q10. The product of two successive integral multiples of 5 is 300. Then the numbers are ***

(a) 25, 30 (b) 10, 15 (c) 30, 35 (d) 15, 20

SHORT ANSWER TYPE QUESTIONS (2 MARKS)

Q1. One year ago, father's age was 8 times as old as his son and now his age is equal to the square of his son's age. Find the son's age? **

Q2. If $p^2x^2 - q^2 = 0$, then find the value of x? *

Q3. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then what is the value of k? **

Q4. If one root of the equation $4x^2 - 2x + p - 4 = 0$ be the reciprocal of other, then what is the value of p? **

Q5. What is the value of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots?*

Q6. Find the roots of the quadratic equation $x^2 - 3x = 0$ *

Q7. Find the sum of the roots of the quadratic equation: $3x^2 - 9x + 5 = 0$? **

Q8. Solve the following quadratic equation for x: $\sqrt{3}\, x^2 + 10x + 7\sqrt{3} = 0$ **

Q9. The product of Riana's age (in years) 5 years ago and his age 7 years from now, is one more than twice his present age. Find their present age? **

Q10. For what value of p for equation $2x^2 + 3x + p = 0$ will have real roots? **

Q11. The sum of a number and its reciprocal is $\frac{5}{2}$. Find the numbers? ***

Q12. What is the discriminant of the quadratic equation $7\sqrt{3}x^2 + 10x - \sqrt{3} = 0$? *

Q13. If a and b are the roots of the equation $x^2 + ax + b = 0$ then what is the value of $a + b$? *

SHORT ANSWER TYPE QUESTIONS (3 MARKS)

Q1. Find the nature of the roots of the following quadratic equations. If the real roots exist, find them: $2x^2 + 4x - 8 = 0$ *

Q2. Using the quadratic formula, solve the following quadratic equation for x

$$p^2x^2 + (p^2 - q^2)\, x - q^2 = 0$$ **

Q3. If α and β are the roots of the equation $2x^2 - 6x + a = 0$ and $2\alpha + 5\beta = 12$,

find the value of a. ***

Q4. If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation

$(x^2 + x) + k = 0$ has equal roots, then find the value of k. ***

Q5. Find the positive value of k for which the equation $x^2+kx+64 = 0$ and $x^2 - 8x+k = 0$ will both have real roots? **

Q6. The sum of ages (in years) of a son and his father is 35 years and product of their ages is 150 years, find their ages. **

Q7. The sum of the squares of two consecutive natural numbers is 421. Find the numbers.***

Q8. A passenger train takes 2 hours less for a journey of 300 km if its speed is increased by 5 km/hr. from its usual speed. Find the usual speed of the train? ***

Q9. Speed of a boat in still water is 11 km/hr. It can go 12 km upstream and return downstream to the original point in 2-hour 45 min. Find the speed of the stream? ***

Q10. A takes 6 days less than the time taken by B to finish a piece of work. If both A and B together can finish it in 4 days, find the time taken by B to finish the work? ***

LONG ANSWER TYPE QUESTIONS (4 MARKS)

Q1. Seven years ago, Rahul's age was five times the square of Rena's age. Three years hence, Rena's age will be two fifth of Rahul's age. Find their present ages. **

Q2. The diagonal of a rectangular field is 16metres more than the shorter side. If the longer side is14 metres more than the shorter side, then find the length of the sides of the field. ***

Q3. One fourth of a herd of camel was seen in the forest. Twice the square root of the herd had gone to the mountains and the remaining 15 camels were seen on the bank of the river. Find the total number of camels. ***

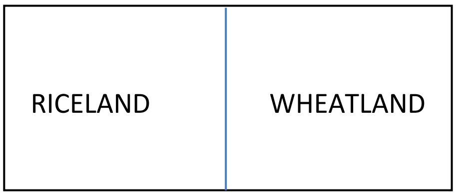
Q4. A train travels 180 km at a uniform speed. If the speed had been 9km/ hour more, it would have taken 1 hour less. Find the speed of the train. ***

Q5. Rs 9000 were divided equally among certain number of persons. Had there been 20 more persons, each would have got Rs160 less. Find the original number of persons. ***

Q6. Two taps running together can fill a tank in 3 and 1/13 hours. If one tap takes 3 hours more than the other to fill the tank, then how much time will each tap take to fill the tank. ***

Q7. A motor boat whose speed is 24 km/ hour in still water takes 1 hour more to go 32 km upstream than to return downstream to the same spot. Find the speed of the stream. ***

Q8. Madhav has a field with total area 1260 square metre. He uses it to grow wheat and rice. The land used to grow wheat is rectangular in shape while the rice land is in the shape of a square as shown in the following figure. The length of wheat land is 3m more than twice the length of Rice land. Find the area of wheat land. **



CASE STUDY BASED QUESTIONS (4 MARKS)

CASE STUDY 1

John and Jayant are very close friends. They decided to go to Ranikhet with their families in separate cars. John's car travels at a speed of x km/hr while Jayant's car travels 5km/hr faster than Johan's car. Johan took 4 hours more than Jayant to complete the journey of 400 km

1. The distance covered by Jayant's car in two hours is

a) 2(x+5) km b) (x-5) km c) 2(x+10) km d) (2x+5) km

2. The quadratic equation describing the speed of John's car is

a) x^2 -5x-500=0 b) x^2+4x-400=0 c) x^2+5x-500=0 d) x^2 -4x+400=0

3. The speed of John's car in km/hr.

a) 20 b) 15 c) 25 d) 10

4. The speed of Jayant's car in km/hr.

a) 25 b) 20 c) 30 d)15

5. Time taken by Jayant to travel 400 km is

a) 20 hours b) 40 hours c) 25 hours d) 16 hours

CASE STUDY 2

The speed of a motor boat is 20 km/hr. For covering the distance of 15 km the boat took 1 hour more for upstream than downstream.

1. If the speed of the stream be x km/hr. then speed of the motorboat in upstream will be

a)20 km/hr b) (20+x) km/hr c)(20-x) km/hr d)(x-20) km/hr

2. If the speed of stream is 10 km/hr, and then the speed of the motor boat in downstream is

a) (20+x) km/hr b) (x-20) km/hr c) 20x km/hr d) 20 x km/hr

3. The quadratic equation giving the speed of current is

a) $x^2+30x-200=0$ b) $x^2+20x-400=0$ c) $x^2+30x-400=0$ d) $x^2-20x-400=0$

4. The speed of current is

a) 20km/hr b) 10 km/hr c)15km/hr d)25km/hr

5. Time taken by the motor boat to cover 15 km upstream is

a) 1 hour b)1.5 hours c) 2 hours d) 3 hours

CBSE CLASS X -PREVIOUS YEAR QUESTIONS FROM QUADRATIC EQUATIONS

Q1. 1. The roots of the quadratic equation $x^2 – 0.04 = 0$ are **[CBSE OD, Set 1, 2020]**

(a) ± 0.2 (b) ± 0.02 (c) 0.4 (d) 2

Q2. If Ritu were younger by 5 years than she really is, then the square of her age would have been 11 more than 5 times of her present age. Find her present age. **[CBSE term2 SQP 2022]**

Q3. Find the nature of roots of the quadratic equation $2x^2 – 4x + 3 = 0$. **[CBSE OD, Set 1, 2019]**

Q4. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real? **[CBSE Delhi, Set 1, 2019]**

Q5. Find the value of k for which the roots of the equation $3x^2 − 10x + k = 0$ are reciprocal of each other. **[CBSE Delhi, Set 1, 2019]**

Q6. If x = 3 is one root of the quadratic equation $x^2 − 2kx − 6 = 0$, then find the value of k. **[CBSE 2018]**

Q7. If the quadratic equation $px^2 − 2\sqrt{5}\, px + 15 = 0$, has two equal roots then find the value of p**.**

[CBSE OD, Term 2, Set 1, 2015]

Q8. If $x = -\frac{1}{2}$, is a solution of the quadratic equation $3x^2 + 2kx − 3= 0$, find the value of k.

[CBSE Delhi, Term 2, Set 1, 2015]

Q9. Find the value of p, for which one root of the quadratic equation $px^2 − 14x + 8 = 0$ is 6 times the other. **[CBSE OD, Term 2, Set 1, 2017]**

Q10. Find the value of k for which the equation $x^2 + k (2x + k − 1) + 2 = 0$ has real and equal roots.

[CBSE Delhi, Term 2, Set 1, 2017]

Q11. Find the roots of the quadratic equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$.

[CBSE Delhi, Term 2, Set 1, 2017]

Q12. Solve for x: $\sqrt{2x+9} + x = 13$ **[CBSE OD, Term 2, Set 2, 2016]**

Q13. If −5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation

$p(x^2 + x) + k = 0$ has equal roots, find the value of k. **[CBSE OD, Term 2, Set 1, 2016]**

Q14. If $x = 2/3$ and $x = -3$ are roots of the quadratic equation $ax^2 + 7x + b = 0$, find the values of a and b. **[CBSE Delhi, Term 2, Set 1, 2016]**

Q15. Solve the following quadratic equation for x:

$$x^2 - 2ax - (4b^2 - a^2) = 0$$

[CBSE OD, Term 2, Set 3, 2015]

Q16. In a flight of 600 km, an aircraft was slowed down due to bad weather. The average speed of the trip was reduced by 200 km/hour and the time of flight was increased by 30 minutes. Find the duration of flight**.** **[CBSE OD, SET 1, 2020]**

Q17. A fast train takes 3 hours less than a slow train for a journey of 600 km. If the speed of the slow train is 10 km/h less than that of the fast train, find the speed of each train. **[CBSE OD, Set 2, 2020]**

Q18. A train covers a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

[CBSE Delhi, Set 2, 2020]

Q19. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream. **[CBSE Delhi, Set 3, 2020]**

Q20. Solve for x:

$$x^2 + 5x - (a^2 + a - 6)$$ **[CBSE OD, Set 2, 2019]**

Q21. A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find Its usual speed.

[CBSE 2018]

Q22. If the roots of the quadratic equation $(a - b)x^2 + (b - c)x + (c - a) = 0$ are equal,

Prove that $2a = b + c$. **[CBSE OD, Term 2, Set 2, 2016]**

ANSWER KEY CHAPTER-4 QUADRATIC EQUATION

VERY SHORT ANSWER TYPE QUESTIONS (1 MARK)

Q.NO.	ANS	Q.NO.	ANS
1	b	6	b
2	a	7	d
3	b	8	a
4	b	9	a
5	c	10	d

SHORT ANSWER TYPE (2 MARKS)

Q.NO	ANS	Q.NO.	ANS
1	7years and 49years	8	$-7/\sqrt{3}$ or $-\sqrt{3}$
2	$+q/p$ or $-q/p$	9	6years
3	K=2	10	$p \leq 9/8$
4	P=8	11	2 and ½
5	K=0 or 8	12	184
6	X=0 or 3	13	a+b= -1
7	Sum=3		

SHORT ANSWER TYPE (3 MARKS)

Q.NO.	ANS	Q.NO.	ANS
1	$X = -1+\sqrt{5}$ & $x = -1-\sqrt{5}$	6	Father-30 years; son-5 years
2	$x = q^2/p^2$, $x = -1$	7	14 & 15
3	a=4	8	Usual speed 25 km/hr
4	k=7/4	9	Speed of stream=5 km/hr
5	k=16	10	No. of days taken by B=12

LONG ANSWER TYPE (3 MARKS)

Q.NO	ANS	Q.NO	ANS
1	Rahul's age=27 years Reena's age=9 years	5	Number of persons=25
2	10m and 24m	6	Larger tap=5 hours Smaller tap=8 hours
3	Total number of camels=36	7	Speed of stream=8 km/hr
4	Speed of train=36km/hr	8	Area=$860m^2$

CASE STUDY BASED QUESTIONS

CASE STUDY 1	CASE STUDY 2
1. 2(x+5) km	1. (20-x) km/hr
2. c	2. (20+x) km/hr
3. 20 km/hr	3 .c
4. 25 km/hr	4. 10
5.16 hours	5. 1.5 hours

PREVIOUS YEAR QUESTIONS

Q.NO	ANS	Q.NO	ANS
1.	x=0.2, -0.2	12	X=8
2	Age = 14 Years	13	K=$\frac{7}{4}$
3	D= -8<0. So, roots will be imaginary.	14	a=3, b= -6
4	K ≤4	15	X=a-2b Or, X=a+2b
5	K=3	16	Original speed of aircraft =600 Km/hr Original Duration of flight = 1hr Increased duration of flight =1.5 hr
6	K=$\frac{1}{2}$	17	Speed of slow train = 40km/hr Speed of fast train = 50 km/hr
7	P=3	18	Original speed of Train =40 km/hr
8	K=- $\frac{9}{4}$	19	Speed of stream = 4 km/hr Speed of boat in still water = 6 km/hr
9	P=3 (P=0 Does not satisfy given condition)	20	X=a-2 or X=-(a+3)
10	K=2	21	Usual speed of plane = 500 km/hr
11	-5/√2 & -√2		

******

CHAPTER -5

ARITHMETIC PROGRESSIONS

Prepared by- RITESH RANJAN
TGT (MATHS)
KV HINOO 1ST SHIFT,RANCHI

KEY CONCEPT

Arithmetic Progression is a sequence of numbers in order, in which the difference between any two consecutive numbers is a constant value. In other words We can say that an arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term. This fixed number is called the common difference of the AP. **Remember that common difference can be positive, negative or zero.**

Example: Consider the following lists of numbers :

(i) 1, 2, 3, 4, . . . (ii) 100, 70, 40, 10, . . . (iii) 3, 3, 3, 3, . . .

We can clearly see that

Common difference in (i) is 1 which is positive.

Common difference in (ii) is -30 which is negative.

Common difference in (iii) is zero

Some daily life Examples of A.P:

(a) The heights (in cm) of some students of a school standing in a queue in the morning assembly are 147 , 148, 149, . . ., 157.

(b) The minimum temperatures (in degree Celsius) recorded for a week in the month of January in a city, arranged in ascending order are – 3.1, – 3.0, – 2.9, – 2.8, – 2.7, – 2.6, – 2.5

Note that in examples given above, there are only a finite number of terms. Such an AP is called a finite AP. Also note that each of these Arithmetic Progressions (APs) has a last term.

General form of an A.P:-

a, a + d, a + 2d, a + 3d, . . . represents an arithmetic progression where **a** is the first term and **d** the common difference. This is called the general form of an AP.

nth Term of an AP:

nth term a_n of the AP with first term a and common difference d is given by

$$\boxed{a_n = a + (n - 1)\,d}$$

a_n is also known as last term of a finite A.P and sometimes It is denoted by l.

Sum of First n Terms of an AP :

- Sum of the first n terms of an AP is given by $\boxed{S_n = \frac{n}{2}[2a + (n-1)d]}$
- We can also write this as

  $S_n = \frac{n}{2}[a + a + (n-1)d]$ i.e., $\boxed{S_n = \frac{n}{2}(a + a_n)}$
- **Now, if there are only n terms in an AP, then $a_n = l$ (the last term).**

  We see that . $\boxed{S_n = \frac{n}{2}(a + l)}$

This form of the result is useful when the first and the last terms of an AP are given and the common difference is not given.

Arithmetic Progression Formula (AP Formulas)

For the first term 'a' of an AP and common difference'd', given below is a list of arithmetic progression formulas that are commonly used to solve various problems related to AP:

- **Common difference of an AP: $d = a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... = a_n - a_{n-1}$**
- **n^{th} term of an AP: $a_n = a + (n - 1)d$**
- **Sum of n terms of an AP: $S_n = n/2\ (2a + (n - 1)\ d) = n/2\ (a + l)$, where l is the last term of the arithmetic progression.**
- **Sum of first n natural numbers= $\frac{n}{2}(n+1)$**
- **Sum of first n odd numbers= n^2**
- **$a_n = S_n - S_{n-1}$**

NOTE:

- **Three consecutive terms of an A.P should be taken as (a-d), a and(a+d)**
- **Four consecutive terms of an A.P should be taken as (a – 3d), (a – d), (a + d) and (a +3d).**
- **Five consecutive terms of an A.P should be taken as (a – 2d), (a – d), a,(a + d) and (a + 2d).**

SECTION A (Solved MCQ)

1. The n^{th} term of an A.P. 5, 2, -1, -4, -7 … is

(a) 2n + 5 (b) 2n – 5 (c) 8 – 3n (d) 3n – 8

Answer: c
Explanation: Here a = 5, d = 2 – 5 = -3
$a_n = a + (n – 1)d = 5 + (n – 1) (-3) = 5 – 3n + 3 = 8 – 3n$

2. The sum of first n odd natural numbers is

(a) $2n^2$ (b) 2n + 1 (c) 2n – 1 (d) n^2

Answer: d
Explanation: Required Sum = 1 + 3 + 5 + … + upto n terms.
Here a = 1, d = 3 – 1 = 2
Sum = $\frac{n}{2}[2 \times 1 + (n – 1) \times 2] = \frac{n}{2}[2 + 2n – 2] = \frac{n}{2} \times 2n = n^2$

3. The 10th term from the end of the A.P. 4, 9,14, …, 254 is

(a) 209 (b) 205 (c) 214 (d) 213

Answer: a
Explanation: Here l = 254, d = 9 - 4 = 5
$\therefore 10^{th}$ term from the end = l – (10 – 1)d = 254 -9d = 254 - 9(5) = 254 – 45 = 209

4. If 2x, x + 10, 3x + 2 are in A.P., then x is equal to

(a) 0 (b) 2 (c) 4 (d) 6

Answer: d
Explanation : Since 2x, x + 10 and 3x + 2 are in A.P.
$\therefore (x + 10)-2x = (3x + 2)-(x+10)$

$\Rightarrow (x + 10)+ (x + 10)= (3x + 2)+2x$
$\Rightarrow 2x + 20 = 5x + 2$
$\Rightarrow 2x – 5x = 2 – 20$
$\Rightarrow 3x = 18$
$\Rightarrow x = 6$

5. 30th term of the A.P: 10, 7, 4… is

(a) 97 (b) 77 (c) -77 (d) -87

Answer: c
Explanation $a_{30} = 10+(30-1)(-3)$

$a_{30} = 10+ (29)(-3)$

$a_{30} = 10-87 = -77$

Practice Questions:

1. Which term of the A.P. 3, 8, 13, 18 … is 78?

(A) 12^{th} (B) 13^{th} (C) 15^{th} (D) 16th

2. In an AP, if d = -4, n = 7, $a_n = 4$, then a is equal to

(A) 6 (B) 7 (C) 20 (D) 28

3. In an AP, if a = 3.5, d = 0,n = 101, then a_n will be
(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

4. The 11th term of the AP: -5, −52 , 0, 52,…
(A) -20 (B) 20 (C) -30 (D) 30

5. Which term of the AP: 21, 42, 63, 84,… is 210?
(A) 9^{th} (B) 10^{th} (C) 11^{th} (D) 12^{th}

6. The 4th term from the end of the AP: -11, -8, -5, 49 is
(A) 37 (B) 40 (C) 43 (D) 58

7. The sum of first 16 terms of the AP: 10, 6 2, … is
(A) -320 (B) 320 (C) -352 (D) -400

8. The sum of first five multiples of 3 is
(A) 45 (B) 55 (C) 65 (D) 75

9. The list of numbers – 10, – 6, – 2, 2,... is

(A) an AP with d = – 16 (B) an AP with d = 4

(C) an AP with d = – 4 (D) not an AP

10. The sum of first 100 multiples of 3 is

(A) 30300 (B) 15150 (C) 300 (D) none of these

SECTION-B SA I (2 Marks Questions)

1. If 17th term of an A.P. exceeds its 10th term by 7. Find the common difference.(**)

Solution: $a_n = a+(n-1)d$

Given, $a_{17} - a_{10} = 7$

$\Rightarrow (a+16d)-(a+9d) = 7$

$\Rightarrow 7d = 7$

$\Rightarrow d = 1$

Therefore, the common difference is 1.

2. Which term of the AP : 3, 9, 15, 21, . . . , is 99?(***)
Solution.- Here, a = 3, d = 9 – 3 = 6
We know that $a_n = a + (n - 1)d$
Let $a_n = 99$ or, $a + (n - 1)d = 99$
$\Rightarrow 3 + (n - 1)6 = 99$
$\Rightarrow (n - 1)6 = 99 - 3 = 96$
$\Rightarrow n - 1 = 16$
$\Rightarrow n = 17.$

3. **Determine the AP whose 3rd term is 5 and the 7th term is 9. (***)**

Solution-We have $a_3 = a + (3 – 1) d = a + 2d = 5$ (1)

and $a_7 = a + (7 – 1) d = a + 6d = 9$ (2)

Solving the pair of linear equations (1) and (2), we get a = 3, d = 1 Hence, the required AP is 3, 4, 5, 6, 7, . . .

4. Find the 11th term from the last term (towards the first term) of the AP : 10, 7, 4, . . ., – 62. (***)

Solution : Here, a = 10, d = 7 – 10 = – 3, l = – 62,
where l = a + (n – 1) d
To find the 11th term from the last term, we will find the total number of terms in the AP.
So, – 62 = 10 + (n – 1)(–3)
i.e., – 72 = (n – 1)(–3) i.e., n – 1 = 24 or n = 25
So, there are 25 terms in the given AP.
The 11th term from the last term will be the 15th term.

$a_{15} = 10 + (15 – 1)(–3) = 10 – 42 = – 32$ i.e., the 11th term from the last term is – 32.

Alternative Solution : If we write the given AP in the reverse order, then a = – 62 and d = 3 So, the question now becomes finding the 11th term with these a and d. So,

$a_{11} = – 62 + (11 – 1) × 3 = – 62 + 30 = – 32$ So, the 11th term, which is now the required term, is – 32.

5. Find the sum of the first 22 terms of the AP : 8, 3, –2, . . . (**)

Solution : Here, a = 8, d = 3 – 8 = –5, n = 22.

We know that $S_n = \frac{n}{2}$ [2a + (n – 1) d]

Putting values of a ,d and n we get S_n= -979

Practice Questions:

1. Find the 20th term from the last term of the AP : 3, 8, 13, . . ., 253. (***)

2. Find the 31^{st} term of an A.P. whose 11^{th} term is 38 and 16^{th} term is 73. (***)

3. Which term of the A.P.84, 80, 76 ………………. is zero? (***)

4. Find the value of x if 3x-4, 4x-7, 7x-3 are in A.P. (***)

5. Which term of the sequence 48, 43, 38, 33……………… is the first negative term? (***)

6. If the 3rd and the 9th terms of an AP are 4 and -8, respectively, then which term of this AP is zero. (***)

7. In a flower bed, there are 23 rose plants in the first row, 21 in the second, 19 in the third, and so on. There are 5 rose plants in the last row. How many rows are there in the flower bed? (**)

8.Find the sum of first 15 multiples of 8. (***)

9. If the sum of the first 14 terms of an AP is 1050 and its first term is 10, find the 20th term. (**)

10. Find the sum of first 1000 positive integers. (**)

11. Which term of the A.P. 3, 8, 13, 18, … is 78? (***)

12. In an AP if the common difference (d) = -4 and the seventh term (a_7) is 4, then find the first term. (**)

SECTION-C (SAII)-3 MARKS QUESTIONS

1. Check whether – 150 is a term of the AP: 11, 8, 5, 2 . . . (**)

Solution: Given AP: 11, 8, 5, 2, …

First term, a = 11

Common difference, $d = a_2 - a_1 = 8 - 11 = -3$

Let −150 be the nth term of this AP.

As we know that

$a_n = a + (n - 1) d$

$\Rightarrow -150 = 11 + (n - 1)(-3)$

$\Rightarrow -150 = 11 - 3n + 3$

$\Rightarrow -164 = -3n$

$\Rightarrow n = 164/3$

Clearly, n is not an integer but a fraction.

Therefore, – 150 is not a term of the given AP.

2. The first term of an A.P. is 5, the common difference is 3, and the last term is 80; find the number of terms. (**)

Solution: Given,a = 5 and d = 3

We know that, the n^{th} term $a_n = a + (n - 1)d$

So, for the given A.P. $a_n = 5 + (n - 1)3 = 3n + 2$

Also given, last term = 80

$\Rightarrow 3n + 2 = 80$

$3n = 78$

$n = 78/3 = 26$

Therefore, there are 26 terms in the A.P.

3. Which term of the AP 121, 117. 113 … is its first negative term? (***)

Solution: Given A.P is 121, 117, 113…

First term (a) = 121, Common difference (d) = 117 – 121 = – 4

We know that, the n^{th} term $a_n = a + (n - 1)d$

Let the n^{th} term is negative, i.e., $a_n < 0$

$121 + (n – 1) – 4 < 0$

$121 + 4 – 4n < 0$

$125 – 4n < 0$

$4n > 125$

$n > 125/4$

$n > 31.25$

The integer which comes after 31.25 is 32.

∴ The 32^{nd} term in the A.P. will be the first negative term.

PRACTICE QUESTIONS:

1. Which term of the sequence 48, 43, 38, 33……………… is the first negative term? (***)

2. Which term of the A.P. 4, 12, 20, 28,…. will be 120 more than its 21^{st} term? (***)

3. Determine the AP whose 5^{th} term is 15 and sum of its 3^{rd} and 8^{th} term is 34. (***)

4. If 19^{th} term of an A.P. is equal to 3 times its 6^{th} term. If the 9^{th} term of the A. P. is 19, find the A.P. (***)

5. If the sum of first n terms of an A.P. is given by $S_n = 4n^2 – 3n$, find the nth term of the A.P. (**)

6. How many terms of the AP 24, 21, 18, . . . must be taken so that their sum is 78? (**)

7. If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms. (**)

8. 7 times the 7^{th} term of an AP is equal to 11 times its 11th term, then what will be its 18th term ? (**)

9. In an AP if a = 1, $a_n = 20$ and $S_n = 399$, then find no.of terms in the given A.P. (*)

10. Find the sum of the first 15 multiples of 8. (***)

11. Find the sum of the first 40 positive integers divisible by 6. (***)

12. Find the sum of all 3 - digit natural numbers which are divisible by 13. (***)

13. Find the sum of all 3- digit natural numbers which are multiples of 11. (***)

14. In an A.P., if the 5^{th} and 12^{th} terms are 30 and 65, respectively, what is the sum of the first 20 terms? (**)

SECTION-D LONG ANSWER TYPE QUESTIONS (5 MARKS)

1. If the 9th term of an A.P. is zero, prove its 29th term is double the 19th term. (**)

Solution: Given, $a_9 = 0$

We know that, the nth term $a_n = a + (n – 1)d$

So, $a + (9 – 1)d = 0 \Rightarrow a + 8d = 0$ ……(i)

Now, The 29th term $a_{29} = a + (29 – 1)d$

$\Rightarrow a_{29} = a + 28d$

And, $a_{29} = (a + 8d) + 20d$ [using (i)]

$\Rightarrow a_{29} = 20d$ ….. (ii)

Similarly, the 19th term $a_{19} = a + (19 – 1)d$

$\Rightarrow a_{19} = a + 18d$

And, $a_{19} = (a + 8d) + 10d$ [using (i)]

$\Rightarrow a_{19} = 10d$ …..(iii)

On comparing (ii) and (iii),

$a_{29} = 2(a_{19})$

Therefore, the 29th term is double the 19th term.

2. If 10 times the 10th term of an A.P. is equal to 15 times the 15th term, show that the 25th term of the A.P. is zero. (**)

Solution: Given,10 times the 10th term of an A.P. is equal to 15 times the 15th term.

We know that, the nth term $a_n = a + (n – 1)d$

$\Rightarrow 10(a_{10}) = 15(a_{15})$

$\Rightarrow 10(a + (10 – 1)d) = 15(a + (15 – 1)d)$

$\Rightarrow 10(a + 9d) = 15(a + 14d)$

$\Rightarrow 10a + 90d = 15a + 210d$

$\Rightarrow 5a + 120d = 0$

$\Rightarrow 5(a + 24d) = 0$

$\Rightarrow a + 24d = 0$

$\Rightarrow a + (25 – 1)d = 0 \Rightarrow a_{25} = 0$

Therefore, the 25th term of the A.P. is zero.

3. Three numbers are in A.P. If the sum of these numbers is 27 and the product 648, find the numbers. (**)

Solution: Let the three numbers of the A.P. be a – d, a, a + d

From the question,

Sum of these numbers = 27

$a - d + a + a + d = 27$

$\Rightarrow 3a = 27$

$\Rightarrow a = 27/3 = 9$

Now, the product of these numbers = 648

$\Rightarrow (a - d)(a)(a + d) = 648$

$\Rightarrow a(a^2 - d^2) = 648$

$\Rightarrow 9(9^2 - d^2) = 648$

$\Rightarrow 9^3 - 9d^2 = 648$

$\Rightarrow 729 - 648 = 9d^2$

$\Rightarrow 81 = 9d^2$

$\Rightarrow d^2 = 9 \Rightarrow d = 3$ or -3

Hence, the terms are 9-3, 9 and 9+3 ⇒ 6, 9, 12 or 12, 9, 6 (for d = -3).

PRACTICE QUESTIONS:

1. Sum of first 14 terms of an A.P is 1050 and its first term is 10. Find its 30th term. (**)
2. Find the 20th term of the AP whose 7th term is 24 less than the 11th term, first term being 12. (**)
3. Which term of the Arithmetic Progression -7, -12, -17, -22,…..will be -82? Is -100 any term of the A.P.? Give a reason for your answer. (**)
4. How many terms of the Arithmetic Progression 45, 39, 33, …. must be taken so that their sum is 180? (***)
5. The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last term to the product of two middle terms is 7:15. Find the numbers. (**)
6. In a certain A.P., the 24th term is twice the 10th term. Prove that the 72nd term is twice the 34th term. (**)
7. The sum of the 4th and 8th terms of an A.P. is 24, and the sum of the 6th and 10th terms is 34. Find the first term and the common difference of the A.P. (***)
8. Find the four numbers in A.P. whose sum is 50 and in which the greatest number is 4 times the least. (**)

(Hint: consider the four terms of the A.P. to be (a – 3d), (a – d), (a + d) and (a + 3d).

9. The first term of an A.P. is 5, the last term is 45, and the sum is 400. Find the number of terms and the common difference. (***)

10. The pth, qth and rth terms of an A.P. are a, b, c, respectively. Show that a(q−r)+b(r−p)+c(p−q)=0. (**)

11. If the ratio of sums of first p terms and q terms of an arithmetic progression is $p^2:q^2$ then find the ratio of its pth and qth term. (**)

SECTION-E (CASE BASED QUESTIONS)-4 MARKS (1+2+1)

1. World Environment Day is an annual global event celebrated on June 05. It is observed to raise awareness on the protection and preservation of the environment. World Environment Day is the largest platform that advocates the practice of sustainable development around the world. With the current pace of the overutilization of natural resources, the future generation is likely to be deprived of these resources. Millions of people around the world participate in this program to protect and conserve the environment to fulfill the needs of the future generation.

In Kendriya Vidyalaya Hinoo , students thought of planting trees in and around the school to reduce air pollution .On the occasion of World Environment Day It was decided that the number of trees, that each section of each class will plant, will be the 10 times as the class, in which they are studying, e.g., a section of Class I will plant 10 trees, a section of Class II will plant 20 trees and so on till Class XII. There are three sections of each class.

Now answer the following questions.

I.	How many trees will be planted by class 7 ?	1
II.	What will be the total no. of trees planted by the students of Kendriya Vidyalaya Hinoo? (OR) Find total no.of trees planted by students of class VI to class XII.	2
III.	Find total no. of trees planted by Primary Section (Class I to V)?	1

2. In the month of April to June 2022, the exports of passenger cars from India increased by 26% in the corresponding quarter of 2021–22, as per a report. A car manufacturing company planned to produce 1800 cars in 4^{th} year and 2600 cars in 8th year. Assuming that the production increases uniformly by a fixed number every year.

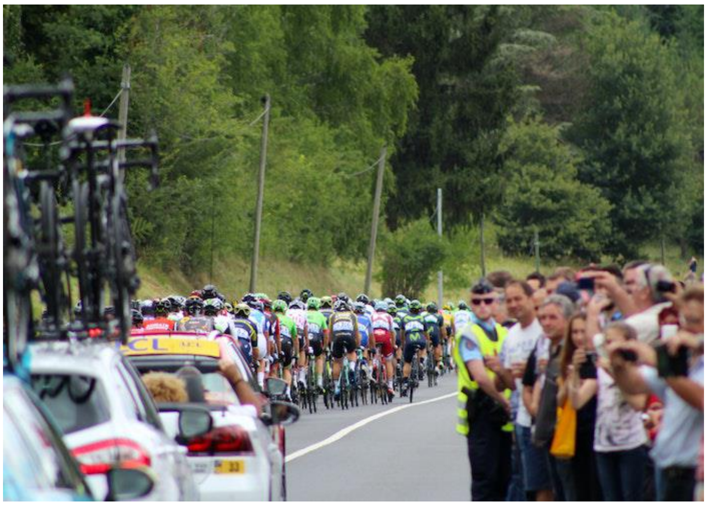
Based on the above information answer the following questions.

I.	Find the production in the 1st year.	1
II.	Find the total production in first 10 years. **[OR]** In how many years will the total production reach 31200 cars?	2
III.	Find the production in the 12th year.	1

3. The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.

I.	If the first circular row has 30 seats, how many seats will be there in the 10th row?	1
II.	For 1500 seats in the auditorium, how many rows need to be there? OR If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row?	2
III.	If there were 17 rows in the auditorium, how many seats will be there in the middle row?	1

4. **Priya is preparing for the Bicycle Marathon.Her racing bicycle has a device to calculate the no.of kilometers she cycled.She decideds to increase the distance she cycles everyday by a fixed number of kilometers.**



I.	On the first day Priya cycled 8 km. In 10 days she cycled a total of total of 170 km. Find fixed no.of kilometer by which she increases her distance everyday.	1
II.	Priya plans to go on a cycle tour from Bangalore to Mangalore covering 425 Km.She travels 20 km on day 1 and increases the distance covered each day by 5 km. In how many days will She reach her destination?	2
III.	Using data of (i) find How many kilometer did She cycle on the third day?	1

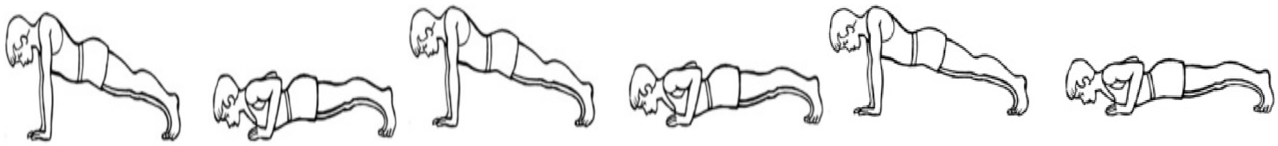
5. Rohit wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, answer the following:

I.	What is the amount paid by him in 30th installment ?	1
II.	What amount does he still have to pay after 30th installment? **OR** If total installments are 40 then what amount is paid in the last installment?	2
III.	What is the total amount paid by him in the 30 installments	1

6. Anuj gets pocket money from his father everyday. Out of the pocket money, he saves Rs. 2.75 on first day, Rs. 3 on second day, Rs. 3.25 on third day and so on.
On the above information, answer the following questions.

I.	what is the amount saved by Anuj on 11^{th} day?	1
II.	what is the amount saved by Anuj on 30^{th} day? OR What is the total amount saved by him in the month of June?	2
III.	what is the amount saved by Anuj in 4 days?	1

7. Push-ups are a fast and effective exercise for building strength. These are helpful in almost all sports including athletics. While the push-up primarily targets the muscles of the chest, arms, and shoulders, support required from other muscles helps in toning up the whole body.



Nitesh wants to participate in the push-up challenge. He can currently make 3000 push-ups in one hour. But he wants to achieve a target of 3900 push-ups in 1 hour for which he practices regularly. With each day of practice, he is able to make 5 more push-ups in one hour as compared to the previous day. If on first day of practice he makes 3000 push-ups and continues to practice regularly till his target is achieved. Keeping the above situation in mind answer the following questions:

I.	Form an A.P representing the number of push-ups per day .	1
II.	Find the total number of push-ups performed by Nitesh up to the day his goal is achieved	2
III.	Find the minimum number of days he needs to practice before the day his goal is accomplished?	1

ANSWER KEY OF PRACTICE QUESTIONS

SECTION-A (MCQ)

1)(D)16^{th} 2)(D)28 3)(B)3.5 4)(B)20 5)(B)10^{TH}

6)(B)40 7)(A) -320 8)(A)45 9)(B)-An AP with d=4 10)(B)15150

SECTION-B (SA I)

1) 158 2) 178 3) 22 4) x= -7/2 5) 11^{th} term

6) 5^{th} term 7) 10 8) 960 9) 200 10) 500500

11) 16^{th} term 12)a=28

SECTION-C (SA II)

1) 11 2) 26 3) a=-1,d=4 AP will be -1,3,7….. 4) a=3,d=2 AP will be 3,5,7…

5) 8n-7 6)n=4,13 7)n^2 8) 0 9) 38 10) 960

11) 4920 12)37674 13) 44550 14)a=10,d=5 Sum=1150

15) radius of first circle=3 cm

Radius second circle=3+4=7 cm

Radius of third circle=7+4=11 cm

Clearly 3,7,11…are in A.P

Let nth circle has radius 43 cm

Then 43=3+(n-1)4

On soving we get n=11

So 11^{th} circle has radius 43 cm.

SECTION-D (LA)

1) a=10,d=10 hence a_{30}=300 2) 126

3) 16^{th} term will be -82.Also on solving n=98/5(not a positive integer)therefore -100 can not be the term of given AP.

4) n=10,6 5) 2,6,10,14 or 14,10,6,2 6) Prove as done in solved examples.

7) a= -1/2,d=5/2 8)5,10,15,20 9) n=16,d=8/3

10) Prove using formula for nth term of an AP 11) 2p-1:2q-1

SECTION-E (CASE BASED QUESTIONS)

1)(i) 70×3=210

(ii)3(10+20+…….120)=2340

(OR)

3(60+70+……120)=1890

(iii) 3(10+20+30+40+50)=450

2) (i) Since the production increases uniformly by a fixed number every year, the no.of Cars manufactured in 1st, 2nd, 3rd, . . .,years will form an AP. So, a + 3d = 1800 & a + 7d = 2600
So a = 1200 & d = 200

(ii) S_n =n/2(2a + (n-1)d
⇒S_{10} =10/2(2×1200 + (10-1)200)=21000

(OR)

S_n =n/2(2a + (n-1)d
⇒31200=n/2(2×1200 + (n-1)200)
⇒31200=n/2(2400 + 200n-200)
⇒31200=n/2(2200 + 200n)
⇒312=n(11 + n) ⇒n^2+11n-312=0
⇒ n^2+24n-13n-312=0⇒(n-13)(n+24)=0⇒n=13 or -24
n=-24(not possible) therefore n=13.

(iii) a_{12}= a+11d=1200+11×200=1200+2200=3400

3) (i) Since each row is increasing by 10 seats, so it is an AP with first term a= 30, and d=10.
So number of seats in 10th row = a_{10} = a+ 9d
= 30 + 9×10 = 120

(ii) S_n =n/2(2a + (n-1)d

⇒1500 = n/2(2 × 30 + (n-1)10)

⇒3000 = 50n + $10n^2$
⇒n^2 +5n -300 =0
⇒n^2 + 20n -15n – 300 =0 ⇒ (n+20) (n-15) =0
⇒n=-20 or n=15
Rejecting the negative value, n= 15

OR

No. of seats already put up to the 10th row = S_{10} = 10/2 {2 × 30 + (10-1)10)}
= 5(60 + 90) = 750
So, the number of seats still required to be put are 1500 -750 = 750

(iii) If no. of rows =17
then the middle row is the 9th row
a_8 = a+ 8d
= 30 + 80= 110 seats

4. (i) HINT:Here a=8 and S=170 n=10 then d=?
Solve it using sum of first n terms formula and you will get d=2
Fixed distance=2 km

(ii) HINT: Here a=20,d=5.S=425 n=?
On solving you will get $n^2+7n-170=0$
n=10,-17
n=-17(rejected) hence n=10

(iii) $a_3=a+2d=8+2\times2=12$ km.

5) (i)3900
(ii)44500 OR 4900
(iii) 73500

6. a=2.75d=0.25

(i) $T_n=2.75+10x0.25=5.25$
(ii) $S_4=4/2(2x2.75+3x0.25)=12.50$
(iii) $T_{30}=2.75+29x0.25=10$

OR, $s_{30}=30/2(2x2.75+29x0.25)=191.25$

7. (I) (i)A.P is 3000,3005,3010,………3900

(II) Sum of n terms=$\frac{n}{2}(a+l)$

$=\frac{181}{2}(3000+3900)=624450$

(III) A.P is 3000,3005,3010,………3900

Finding n=181

Minimum no.of days before his goal is accomplished=181-1=180

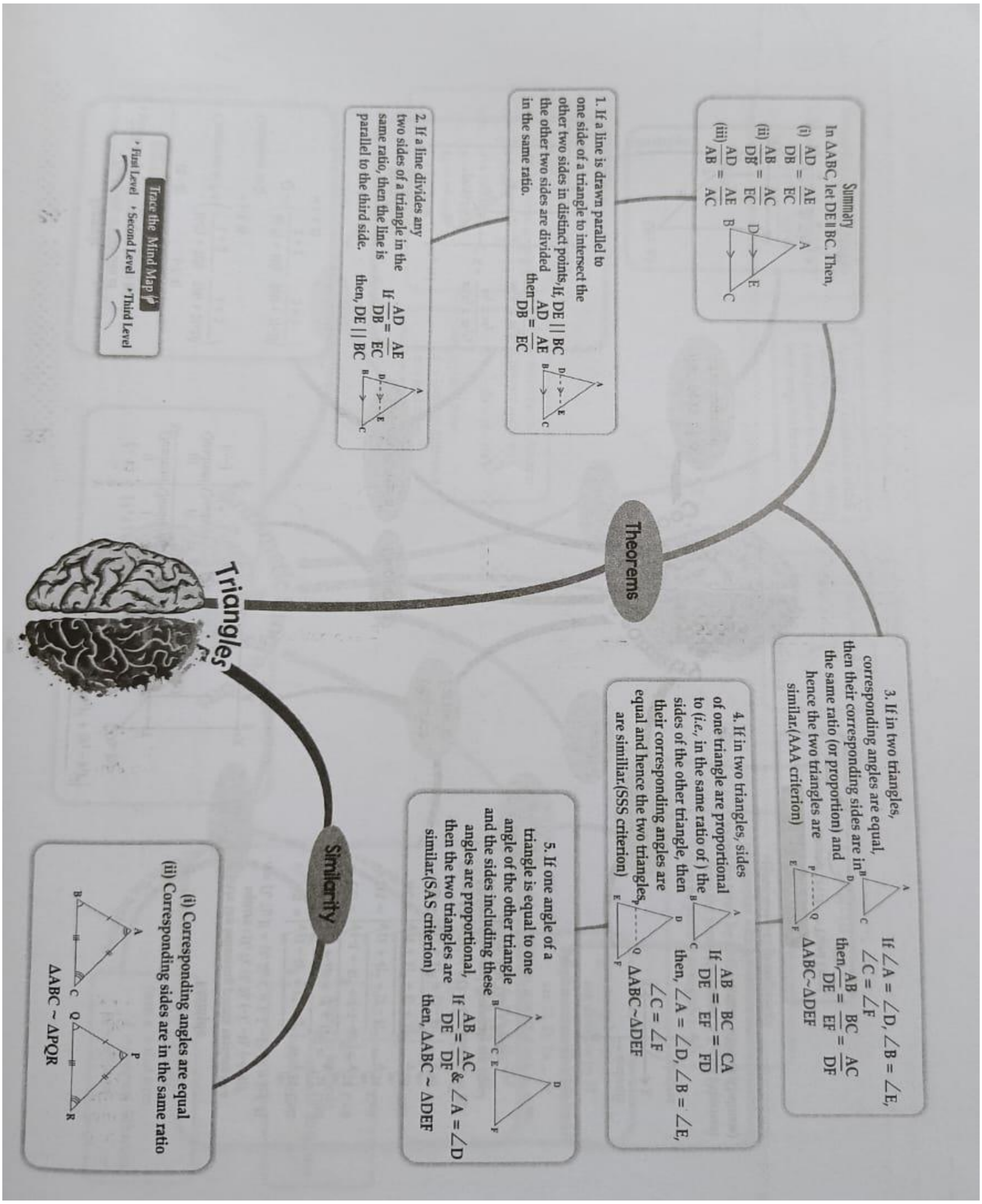
******

CHAPTER -6

TRIANGLES

Prepared by- JAYANTA KUMAR PATAR
TGT (MATHS)
KV MAITHON DAM

MIND MAP



CONCEPT:

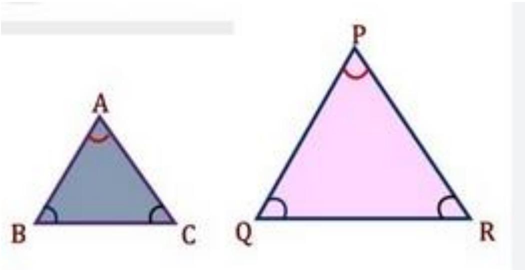
- Two figures having the same shape but not necessarily the same size are called ***similar*** figures.

- Two figures are said to be ***congruent*** if they have the same shape and the same size

Note- *All congruent figures are similar but the similar figures need not be congruent.*

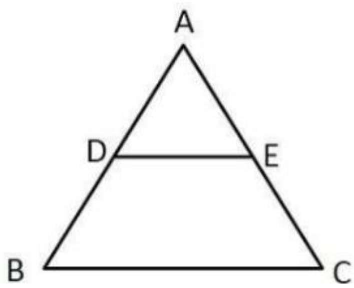
- Two ***polygons*** of the same number of sides are similar, ***if*** (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (i.e., proportion).
- **SIMILAR TRIANGLES**
-

Two triangles are said to be ***similar,*** if
(i) Their corresponding angles are equal and $\angle A=\angle P$, $\angle B=\angle Q$, $\angle C=\angle R$
(ii) Their corresponding sides are in the same ratio (or proportion). AB/PQ =AC/PR = BC/QR



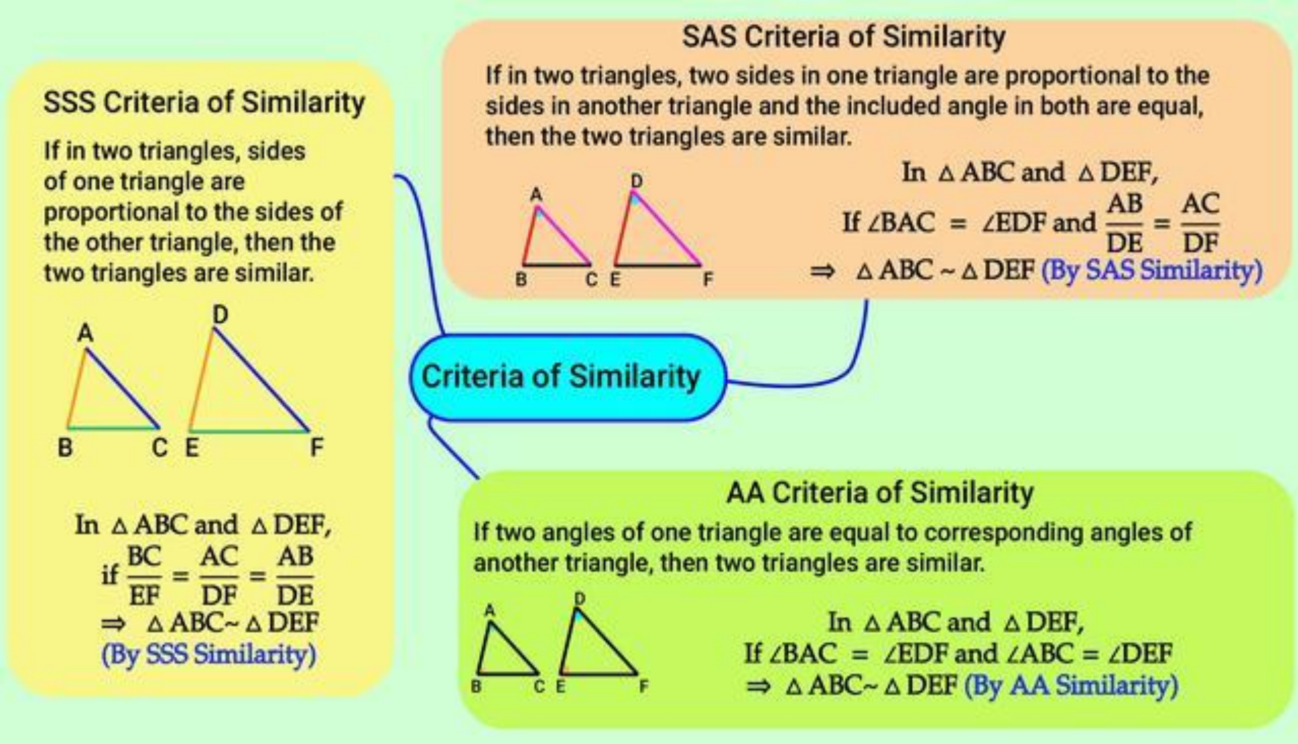
THEOREM:

<u>Basic proportionality Theorem/ Thales Theorem</u>: If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.



If DE// BC, Then $\frac{AD}{DB} = \frac{AE}{EC}$

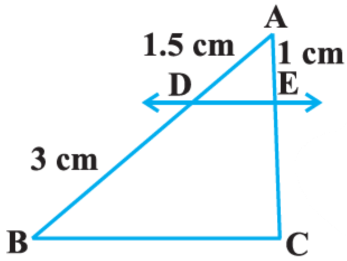
Criteria for similarity of triangles



NOTE: AA similarity criterion can also be stated as AAA similarity criterion.

SOLVED EXAMPLES

*****Q1. If DE || BC. Find EC**



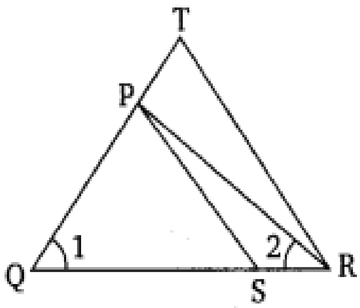
Ans- In ΔABC, DE || BC

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ (By BPT)

$\frac{1.5}{3} = \frac{1}{EC}$

EC $= \frac{3}{1.5} = 2$ cm

****Q2. In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and ∠1 = ∠2. Show that ΔPQS ~ ΔTQR.**



Ans- From the figure ∠1 = ∠2

PQ = PR (Sides opposite to equal angles are equal)

In Δ PQS and Δ TQR

Or $\frac{QR}{QS} = \frac{QT}{PR}$(given)

Or $\frac{QR}{QS} = \frac{QT}{PQ}$ (PQ = PR)

$\angle PQS = \angle TQR$

$\therefore \Delta PQS \sim \Delta TQR$ (By SAS)

***Q3. If AD and PM are medians of triangles ABC and PQR, respectively where ΔABC ~ Δ PQR,**

prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

Ans-

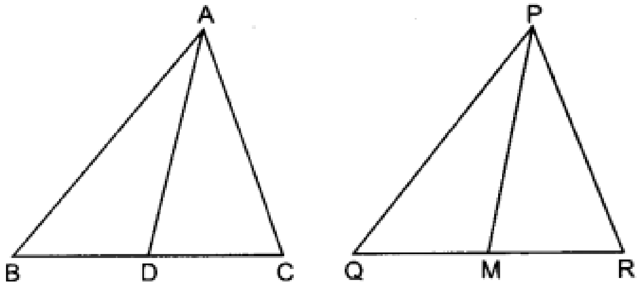
When $\Delta ABC \sim \Delta PQR$

$\Rightarrow \angle ABC = \angle PQR$

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR}$$

$$\frac{AB}{PQ} = \frac{BD}{QM}$$



In ΔABD and ΔPQM,

$$\frac{AB}{PQ} = \frac{BD}{QM}$$ [As proved]

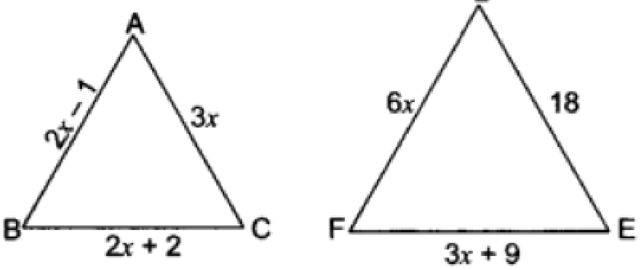
$\angle B = \angle Q$

$\therefore \Delta ABD \sim \Delta PQM$

$$\frac{AB}{PQ} = \frac{AD}{PM}$$ [Corresponding sides of similar triangles]

****Q4.**

In Fig, if ΔABC ~ ΔDEF and their sides are of lengths (in cm) as marked along with them, then find the lengths of the sides of each triangle



Ans- ΔABC ~ ΔDEF (Given)

therefore, $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$

So, $\frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$

Now, taking $\frac{2x-1}{18} = \frac{3x}{6x}$, we have

$$\frac{2x-1}{18} = \frac{1}{2}$$

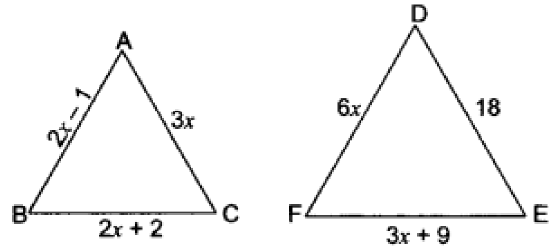


Fig. 7.26

$\Rightarrow 4x - 2 = 18$

$\Rightarrow x = 5$

$\therefore AB = 2 \times 5 - 1 = 9$,

$BC = 2 \times 5 + 2 = 12$

$CA = 3 \times 5 = 15$,

$DE = 18$,

$EF = 3 \times 5 + 9 = 24$ and

$FD = 6 \times 5 = 30$

Hence, AB = 9 cm, BC = 12 cm, CA = 15 cm DE = 18 cm, EF = 24 cm, FD = 30 cm

***Q5. In ΔABC, if $\angle ADE = \angle B$, then prove that $\Delta ADE \sim \Delta ABC$. Also, if AD = 7.6 cm, AE = 7.2 cm, BE = 4.2 cm and BC = 8.4 cm, then find DE.**

Ans-

Given: $\angle ADE = \angle B$, i.e. $\angle 1 = \angle 2$

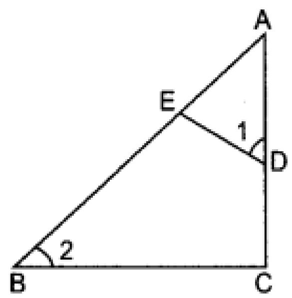
To prove: $\Delta ADE \sim \Delta ABC$

Proof: In ΔADE and ΔABC

$\angle 1 = \angle 2$ [Given]

$\angle A = \angle A$ [Common]

So, $\Delta ADE \sim \Delta ABC$ [By AA similarity]



$$\Rightarrow \quad \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \quad \frac{7.6}{7.2+4.2} = \frac{DE}{8.4} \qquad \{\because AB = AE + BE = 7.2 + 4.2\}$$

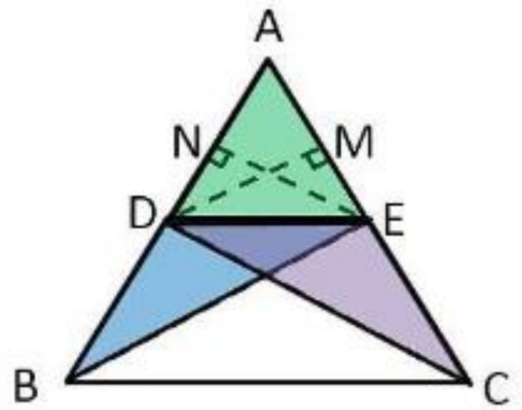
$$\Rightarrow \quad \frac{7.6}{11.4} = \frac{DE}{8.4} \Rightarrow DE = \frac{7.6 \times 8.4}{11.4} = 5.6$$

Hence, DE = 5.6 cm.

*****Q6. State and prove Basic proportionality Theorem.**

Ans-

statement- ***If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.***



Given- A triangle ABC in which DE // BC

To prove- $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join BE and CD and draw DM ⊥ AC and EN ⊥ AB.

Proof:

$$\text{ar (ADE)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times AD \times EN \quad ...(1)$$

$$\text{ar (BDE)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times DB \times EN \quad ...(2)$$

Divide (1) and (2)

$$\frac{\text{ar (ADE)}}{\text{ar (BDE)}} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN}$$

$$\frac{\text{ar (ADE)}}{\text{ar (BDE)}} = \frac{AD}{DB} \quad ...(A)$$

$$\text{ar (ADE)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times AE \times DM \quad ...(3)$$

$$\text{ar (DEC)} = \frac{1}{2} \times \text{Base} \times \text{Height}$$

$$= \frac{1}{2} \times EC \times DM \quad ...(4)$$

Divide (3) and (4)

$$\frac{\text{ar (ADE)}}{\text{ar (DEC)}} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM}$$

$$\frac{\text{ar (ADE)}}{\text{ar (DEC)}} = \frac{AE}{EC} \quad ...(B)$$

Since Δ BDE and ΔDEC are on the same base DE and between the same parallels BC and DE.

Therefore, ar (BDE) = ar (DEC)

Hence from (A), and (B), we have $\frac{AD}{DB} = \frac{AE}{EC}$ Proved.

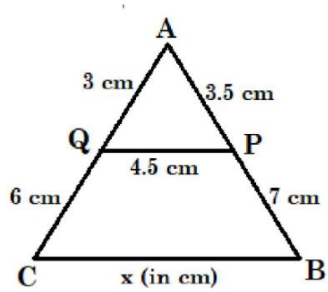
PRACTICE QUESTION

SECTION A (MULTIPLE CHOICE QUESTIONS 1 MARK)

***Q1.** Given that $\Delta ABC \sim \Delta DEF$. If DE = 2AB and BC =3 cm then, EF is equal to ___________.

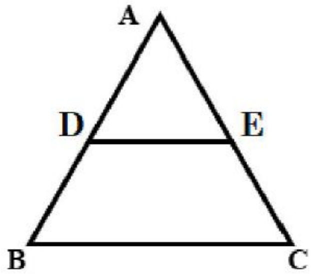
(a) 12 cm (b) 2 cm (c) 1.5 cm (d) 6 cm

****Q2**. In the given figure, write the value of x.



(a) 9 cm (b) 10.5 cm (c) 13.5 cm (d) 12 cm

***Q3**. In the given figure, $\frac{AD}{DB}=\frac{AE}{EC}$ and $\angle ADE = 70^o$, $\angle BAC = 50^o$, then $\angle BCA$ =

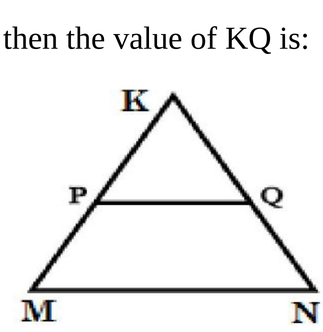


(a) 70º (b) 50º (c) 80º (d) 60º

***Q4.**The shadow of a tower 5 m long is 2 m . At the same time the shadow of a tree 12.5 m high is:

(a) 3 m (b) 3.5 m (c) 5 m (d) 4.5 m

****Q5.**In the figure given below, ,$\frac{KP}{PM} = \frac{4}{13}$, and KN = 20.4 cm, then the value of KQ is:



(a) 2.8 cm (b) 3.8 cm (c) 4.8 cm (d) 5.8 cm

****Q6. If** $\Delta PQR \sim \Delta XYZ$, $\angle Q = 50^o$ and $\angle R = 70^o$ then $\angle X + \angle Y$ is equal to

(a) 70º (b) 50º (c) 120º (d) 110º

Q7. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

(a) ∠B = ∠E (b) ∠A = ∠D (c) ∠B = ∠D (d) ∠A = ∠F

Q8. In ΔABC, D is point on side AB and E is a point on side AC such that ∠ADE = ∠ABC, AD = 2, BD = 3 and AE = 3, then what is the value of CE?

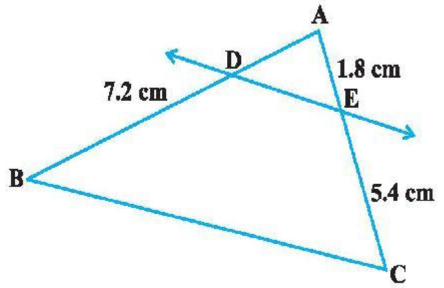
(a) 6 cm (b) 3 cm (c) 4.5 cm (d) 5 cm

*Q9. If in two triangles ABC and DEF $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$ then

(a) ΔFDE ~ ΔCAB (b) ΔFDE ~ ΔABC

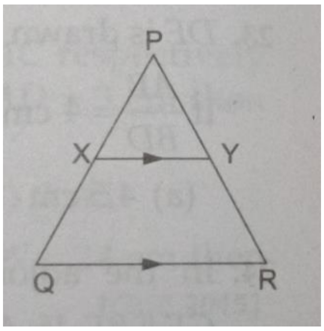
(c) ΔCBA ~ ΔFDE (d) ΔBCA ~ ΔFDE

Q10. In figure DE || BC then the value of AD is



(a) 2 cm (b) 2.4 cm (c) 3 cm (d) none of the above

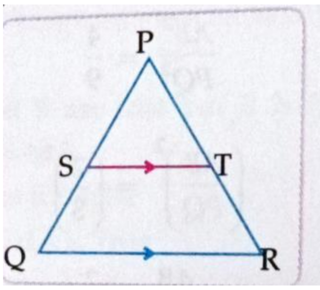
***Q11. In the adjoining figure, XY // QR and PX : XQ =5:6. Then, XY : QR equals



(a)5:11 (b) 6:5 (c) 11:5 (d)11:6

** Q12. In the following figure, ST//QR, Point S divides PQ in the ratio 4:.

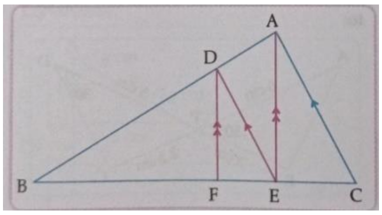
If ST=1.6 cm, what is the length of QR?



(a) 0.71 cm (b) 2 cm (c) 3.6 cm (d) none of the above

13. In the figure below, DE//AC and DF//AE.

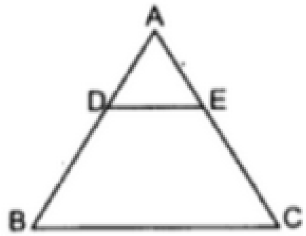
Which of these is equal to $\frac{BF}{FE}$?



(a) DF/AE
(b) BE/EC
(c) BA/AC

(d) FE/EC

Q14. In figure, if DE || BC, AD=3 cm, BD= 4 cm and BC= 14 cm, then DE equals



(a) 7 cm (b) 6 cm (c) 4 cm (d) 3 cm

Q15. D and E are the midpoints of side AB and AC of a triangle ABC, respectively and BC= 6cm.

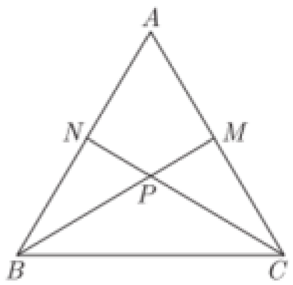
If DE || BC, then the length of DE is:

(a) 2.5 cm (b) 3cm (c)5cm (d) 6cm

SECTION B

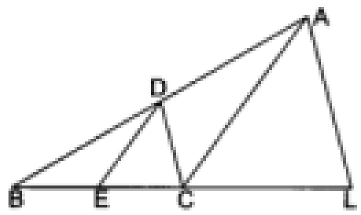
(SHORT ANSWER TYPE QUESTION 2 MARKS)

***Q16.** In the given figure, AM: MC =3:4, BP:PM =3:2 and BN = 12 cm. Then find AN



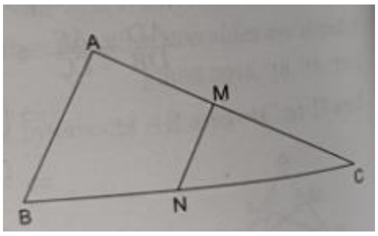
Q17. A street light bulb is fixed on a pole 12 m above the level of the street. If a woman of height 3 m casts a shadow of 6m, what is the length of the shadow of the pole?

Q18. In the given Fig, CD || LA and DE || AC. Find the length of CL, if BE = 4 cm and EC =2 cm.



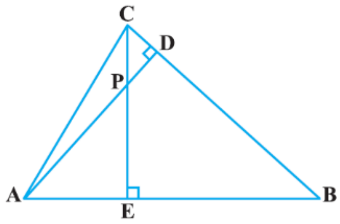
***Q19.** XY is drawn parallel to the base BC of a Δ ABC cutting AB at X and AC at Y. If AB = 4 BX and YC =2cm, then AY is

Q20. In the given figure, MN // AB, BC = 7.5cm, AM = 4cm and MC = 2cm. find the length of BN.



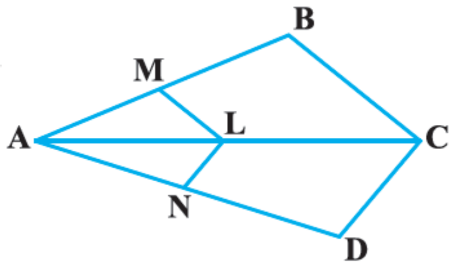
*****Q21.** In Fig., altitudes AD and CE of ΔABC intersect each other at the point P.

Show that: ΔAEP ~ ΔCDP



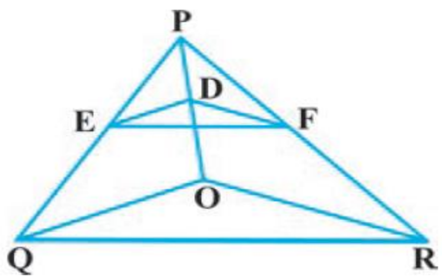
*****Q22.** ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

*****Q23.**In the given figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$



***Q24.**S and T are points on sides PR and QR of Δ PQR such that ∠ P = ∠ RTS. Show that Δ RPQ ~ Δ RTS.

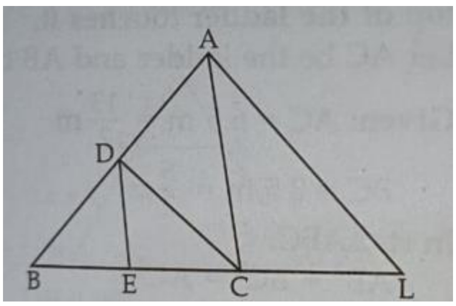
****Q25.** In the given Figure, DE || OQ and DF || OR. Show that EF || QR.



SECTION C

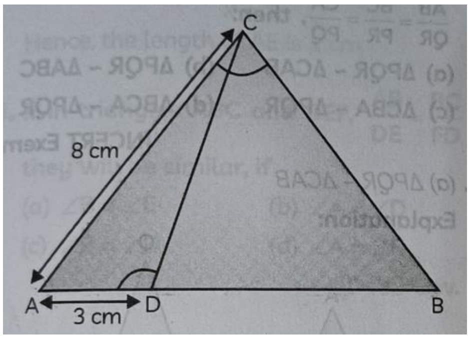
(SHORT ANSWER TYPE QUESTION 3 MARKS)

****Q26.** In the given figure, CD // LA and DE // AC. Find the length of CL if BE = 4cm and EC = 2cm.

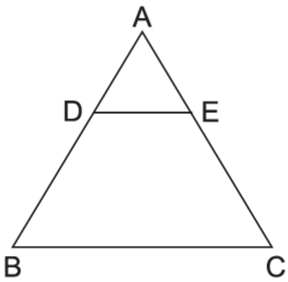


***Q27.** In a ΔABC, DE // BC with D on AB and E on AC. If $\frac{AD}{BD} = \frac{3}{4}$ Find $\frac{BC}{DE}$

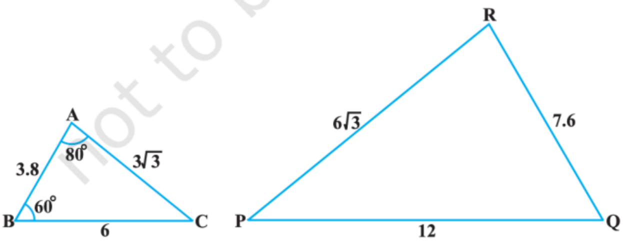
****Q28.** In the given figure, ∠ACB = ∠CDA, AC = 8cm, AD = 3cm, then find BD.



****Q29.** In the given figure DE // BC and $\frac{AD}{BD} = \frac{3}{5}$ if AC = 4.8cm, find the length of AE.

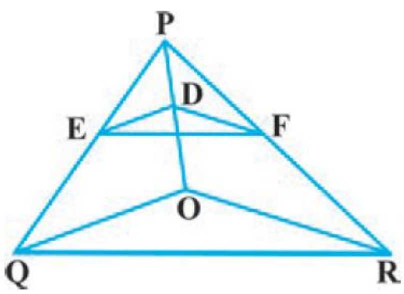


***Q30.** Observe the Figure and then find ∠ P.

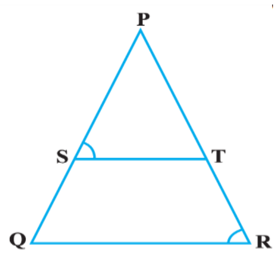


*****Q31.** If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$

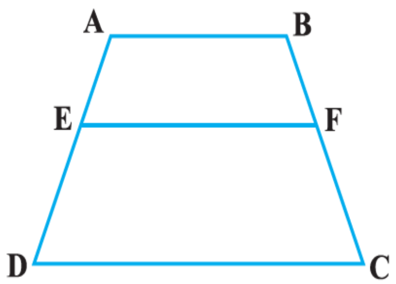
****Q32.** In the given Figure, DE || OQ and DF || OR. Show that EF || QR.



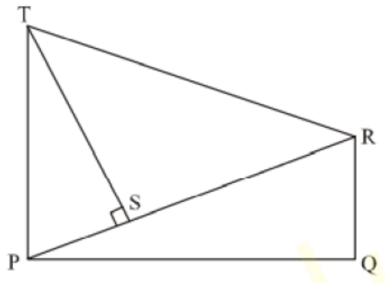
****Q33.** In the given Figure, $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠ PST = ∠ PRQ. Prove that PQR is an isosceles triangle



***Q34.** ABCD is a trapezium with AB || DC. E and F are points on non-parallel sides AD and BC respectively such that EF is parallel to AB. Show that $\frac{AE}{ED} = \frac{BF}{FC}$



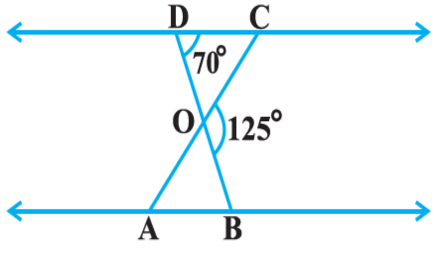
***Q35.** In the given figure, RQ and TP are perpendicular to PQ, also TS perpendicular to PR . Prove that ST.RQ = PS.PQ.



SECTION D

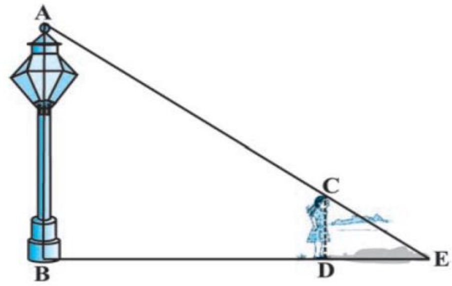
(LONG ANSWER TYPE QUESTIONS 5 MARKS)

****Q36.** In the given figure,Δ ODC ~ Δ OBA, ∠BOC = 125°and ∠ CDO = 70°. Find ∠ DOC, ∠DCO and∠ OAB.



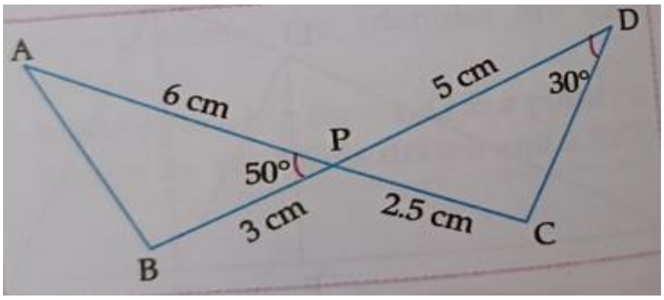
*****Q37.**A vertical pole of length 3 m casts a shadow 2 m long on the ground and at the same time a tower casts a shadow 14 m long. Find the height of the tower.

*****Q38.**A girl of height 90 cm is walking away from the base of a lamp-post at a speed of 1.2 m/s. If the lamp is 3.6 m above the ground, find the length of her shadow after 4 seconds.



****Q39.** Raj wanted to determine the height of a tree on the corner of his block. He knew that a certain fence by the tree was 4 feet tall. At 3 PM, he measured the shadow of the fence to be 2.5 feet tall. Then he measured the tree's shadow to be 11.3 feet. What is the height of the tree?

***Q40.** In the given figure, two line segment AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50º, and ∠CDP= 30º, Then Find the ∠PBA.

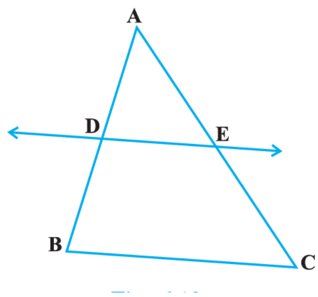


*****Q41.**A 15m high tower casts a shadow 24m long at a certain time and at the same time, a telephone pole casts a shadow 16m long. Find the height of the telephone pole.

*****Q42.** Let ABC be a triangle and D and E be two points on side AB such that AD = BE. If DP // BC and EQ // AC, then prove that PQ // AB.

*****Q43.**State and prove Thales' theorem.

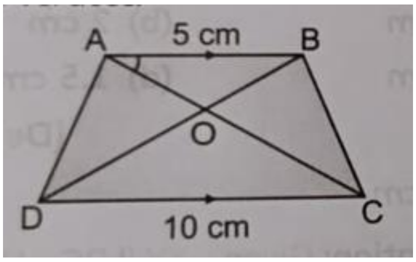
****Q44.** If a line intersects sides AB and AC of a Δ ABC at D and E respectively and is parallel to BC, prove that $\frac{AD}{AB} = \frac{AE}{AC}$



*****Q45.** D is a point on the side BC of a triangle ABC such that ∠ ADC = ∠ BAC. Show that $CA^2 = CB.CD$.

CASE BASED QUESTIOS

****Q46.** A farmer has a field in the shape of a trapezium, whose map with scale 1cm = 20m, is given below: The field is divided into four parts by joining the opposite vertices.



Based on the above information, answer the questions:

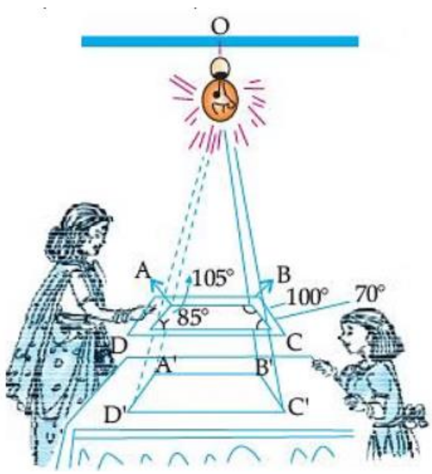
(i) If ABCD is a trapezium in which AB // DC and its diagonals intersect each other at the point O then AO/BO is equal to______

(ii) In a trapezium ABCD, AB // CD, the diagonal AC and BD intersect at O. If OC = 3 cm, OB = 2 cm and OA = 4 cm then find the length of side OD.

(iii) Prove that the two triangular region AOB and COD are similar.

***Q47**. Read the following text and answer the below questions

Seema placed a light bulb at a point O on the ceiling and directly below it placed a table. Now she put a cardboard of shape ABCD between table and light bulb. Then a shadow of ABCD is casted on the table as $A^1B^1C^1D^1$ (See figure). Quadrilateral $A^1B^1C^1D^1$is in an enlargement of ABCD with scale factor 1:2 also AB = 1.5 cm, BC = 25 cm, CD = 2.4cm and AD = 2.1 cm, $\angle$ A=105º, $\angle$B = 100º, $\angle$X = 70º and $\angle$ D = 85º

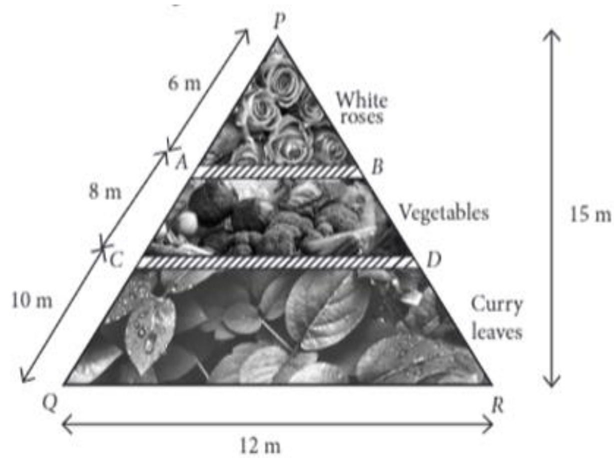


(i) What is the sum of $\angle C^1$and $\angle D^1$

(ii) What is the measure of $\angle A^1$

(iii) What is the length ofA^1B^1

****Q48.** In the backyard of house, Meeta has some empty space in the shape of a ΔPQR. She decided to make it a garden. She divided the whole space into three parts by making boundaries AB and CD using bricks to grow flower ,where AB//CD//QR as shown in figure.



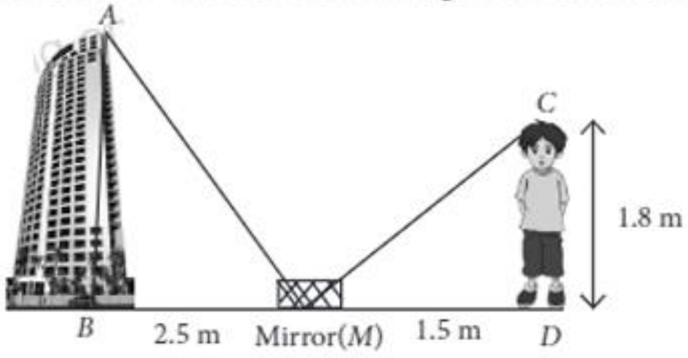
Based on the above information, answer the questions:

(i) What is the length of AB ?

(ii) What is the length of CD ?

(iii) What is the area of whole land?

*****Q49.** Ram is a student of class X, One day his math teacher gave an activity to measure the height of the building. Ram accepted the challenge and places a mirror on ground level to determine the height of building .He is standing at a certain distance so that he can see the top of the building reflected from mirror. Ram's eye level is at 1.8m above the ground. The distance of Ram from the mirror and that of building from mirror are 1.5m and 2.5m respectively.



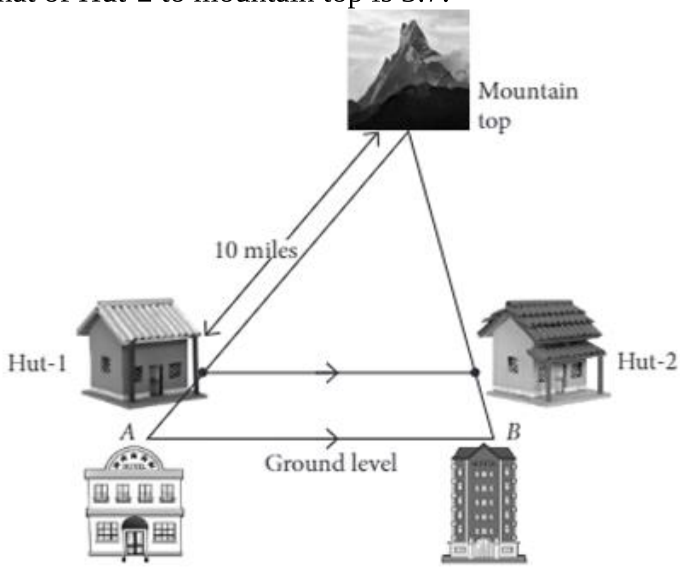
Based on the above information, answer the questions:

(i) Name the triangles which are similar and also mentioned which criterion of similarity is applied here?

(ii) What is the height of the building?

(iii) If Δ ABM and ΔCDA are similar where CD = 6cm, MD = 8 cm and BM = 24cm, then find the length of AB?

****Q50.** Two hotels are at the ground level near to a mountain. On moving a certain distance toward the top of the mountain two huts are situated as shown in the figure. The ratio between the distance from hotel B to Hut-2 and that of Hut-2 to mountain top is 3:7.



Based on the above information, answer the questions:

(i) What is the distance between the hotel A and Hut-1?

(ii) If the horizontal distance between the Hut-1 and Hut-2 is 8miles, then find the distance between two hotels?

(iii) If the distance from mountain top to Hut-1 is 5miles more than that of distance from hotel B to mountain top, then what is the distance between Hut-2 and mountain top?

ANSWER KEY
CHAPTER-6 TRIANGLES
SECTION A

QUESTION	ANSWER	QUESTION	ANSWER	QUESTION	ANSWER
1	D	**6**	D	**11**	A
2	C	**7**	C	**12**	C
3	D	**8**	C	**13**	B
4	C	**9**	A	**14**	B
5	C	**10**	B	**15**	B

SECTION B

QUESTION	ANSWER
16	14 cm.
17	24m
18	3 cm
19	6cm
20	5cm

SECTION C

QUESTION	ANSWER
26	3cm.
27	$-\frac{7}{3}$
28	55/3cm
29	1.8cm
30	$\angle P = 40°$

SECTION D

QUESTION	ANSWER
36	$\angle DOC=55^o$,$\angle DCO=55^o$ and $\angle OAB = 55^o$
37	21m
38	1.6 m long
39	Height of the tree is 18.08 feet
40	100º
41	10m

CASE BASED QUESTIOS

46.(i) CO/DO
(ii) 6cm
(iii) Prove

47.(i) 155^o
(ii) 105^o
(iii) 3cm

48. (i) 3m
(ii) 7cm
(iii) $90m^2$

49. (i)- ΔABM and ΔCDM, AA Criterion.
(ii) 3cm
(iii) 18cm

50. (i) 4.29 miles
(ii) 11.43 miles
(iii) 3.5miles

******

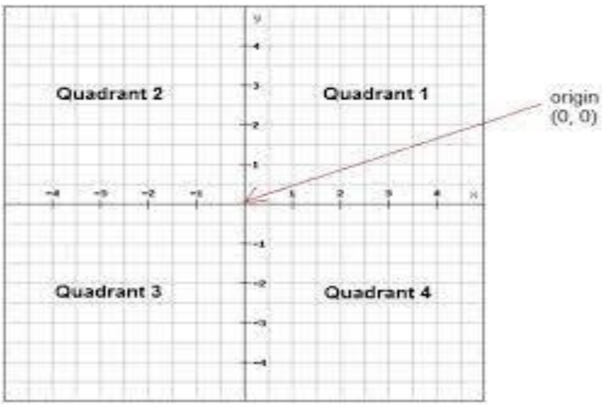
CHAPTER -7

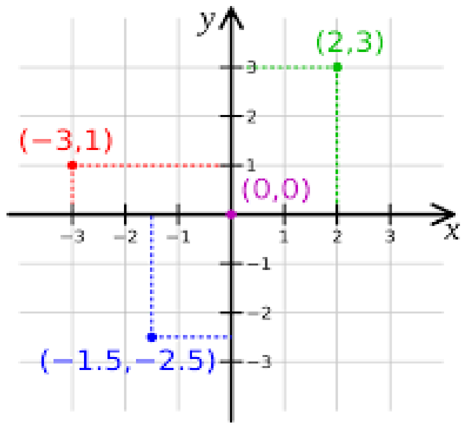
COORDINATE GEOMETRY

Prepared by- Mrs Preety
TGT (MATHS)
KV Bhurkunda (Ranchi)

Important Notes

- Two perpendicular number lines intersecting at origin are called co-ordinate axes. The horizontal line is the X-axis and the vertical line is Y-axis.
- The point of intersection of X-axis and Y-axis is called origin and denoted by O.
- Cartesian plane is a plane obtained by putting the co-ordinate axes perpendicular to each other in the plane. It is also called co-ordinate plane of XY -plane.

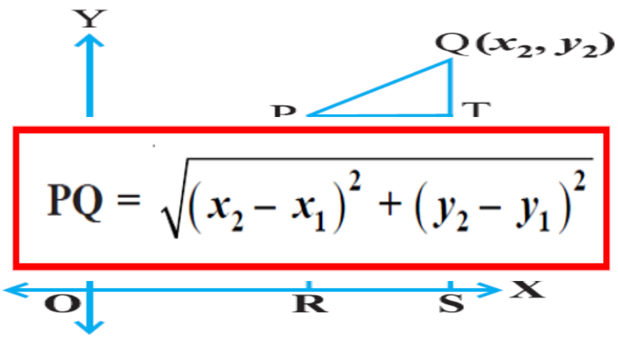




- The axes divide the cartesian plane into four parts called quadrants as shown above.

DISTANCE FORMULA:

- The distance between two points P and Q is calculated by distance formula (using pythagoras theorem in right angled triangle).



$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Problems based on distance formula

• To show that a given figure is a

• Parallelogram – prove that the opposite sides are equal

• Rectangle – prove that the opposite sides are equal and the diagonals are equal.

• Parallelogram but not rectangle – prove that the opposite sides are equal and the diagonals are not equal. • Rhombus – prove that the four sides are equal

• Square – prove that the four sides are equal and the diagonals are equal.

• Rhombus but not square – prove that the four sides are equal and the diagonals are not equal.

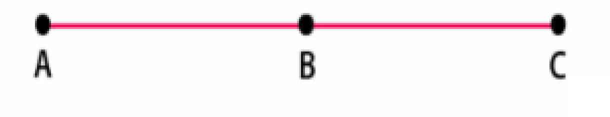
• Isosceles triangle – prove any two sides are equal.

• Equilateral triangle – prove that all three sides are equal. • Right triangle – prove that sides of triangle satisfy Pythagoras theorem.

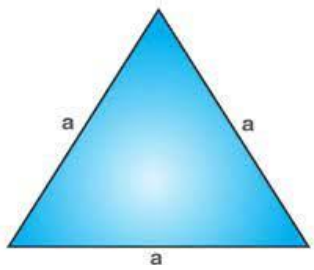
❖ **DISTANCE OF A POINT P (X,Y) FROM ORIGIN.**

Since coordinate of origin is (0,0), Then by applying distance formula, distance from P(x,y) is $\mathbf{OP} = \sqrt{X^2 + Y^2}$

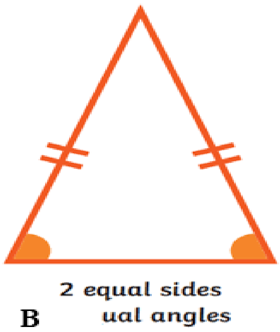
❖ Three points A , B and C are collinear if the distance AB, BC and CA are such that the sum of two distances is equal to the third.



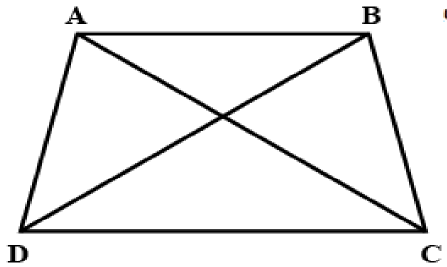
❖ Three points A,B and C are vertices of an equilateral triangle if AB=BC=AC.



❖ Three points A,B and C are vertices of an isosceles triangle if AB=BC or BC=AC or AC=AB.



❖ For the given four points A,B,C and D:



1. If AB=BC=CD=DA; AC=BD , then ABCD is a square.
2. If AB=BC=CD=DA; AC≠BD, Then ABCD is a rhombus.
3. If AB=CD, BC=DA , then ABCD

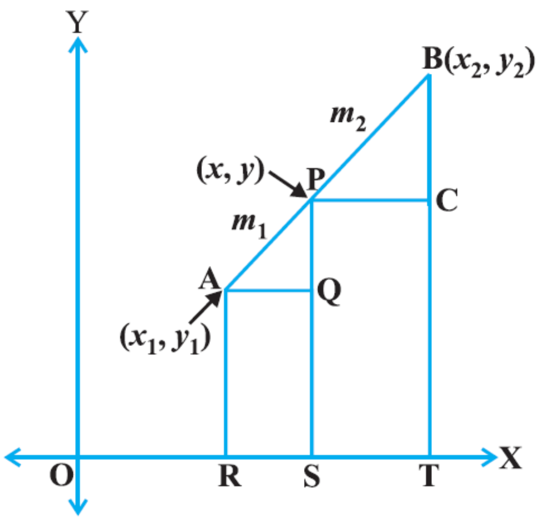
Is a rectangle.

4. If AB=CD,BC=DA ; AC≠BD,
Then ABCD is a parallelogram.

- Diagonals of rhombus, square, rectangle and parallelogram always bisect each other.
- Diagonals of rhombus and square bisect each other at right angles.

SECTION FORMULA:

$$P(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$



- **If the ratio in which P divides AB is *k* : 1, then the coordinates of the point P will be**

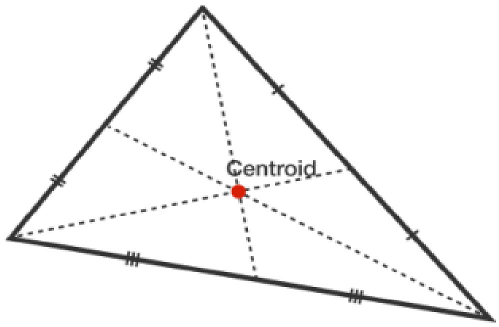
$$P(x, y) = \left(\frac{kx_2 + x_1}{k+1}, \frac{ky_2 + y_1}{k+1}\right)$$

- **Special Case** : The mid-point of a line segment divides the line segment in the ratio 1 : 1. Therefore, the coordinates of the mid-point P of the join of the points A(x_1, y_1) and B(x_2, y_2) is

$$P(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- **CENTROID OF TRIANGLE:** The centroid of a triangle is the center of the triangle. It is referred to as the point of concurrency of medians of a triangle.

The coordinates of the vertices of a triangle are A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3), then centroid C (x, y) of given triangle ABC can be find out using,



$$\boxed{C\ (x, y) = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)}$$

Q. If (a/3, 4) is the mid-point of the segment joining the points P(-6, 5) and R(-2, 3), then the value of 'a' is

(a) 12 (b) -6 (c) -12 (d) -4

Answer: c

Explanation:

Reason: mid-point = $\left(\frac{-6-2}{2}, \frac{5+3}{2}\right) \Rightarrow \left(\frac{a}{3}, 4\right) = (-4, 4) \therefore \frac{a}{3} = -4 \Rightarrow a = -12$

Q..The points (1,1), (-2, 7) and (3, -3) are *

(a) vertices of an equilateral triangle (b) collinear

(c) vertices of an isosceles triangle (d) none of these

Answer: b

Explanation: Reason: Let A(1, 1), B(-2, 7) and C(3, 3) are the given points, Then, we have

$$AB = \sqrt{(-2-1)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$$

$$BC = \sqrt{(3+2)^2 + (-3-7)^2} = \sqrt{25+100} = \sqrt{125} = 5\sqrt{5}$$

$$\text{and } AC = \sqrt{(3-1)^2 + (-3-1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Clearly BC = AB + AC. $\therefore$ A, B, C are collinear.

Lets practice some MCQ's

1. The distance of the point P (2, 3) from the x-axis is

(a) 2 (b) 3 (c) 1 (d) 5

2. The distance between the point P(1, 4) and Q(4, 0) is

(a) 4 (b) 5 (c) 6 (d) $3\sqrt{3}$

3. The points (-5, 1), (1, p) and (4, -2) are collinear if the value of p is

(a) 3 (b) 2 (c) 1 (d) -1

4. The distance of the point (α, β) from the origin is

(a) $\alpha + \beta$ (b) $\alpha^2 + \beta^2$ (c) $|\alpha| + |\beta|$ (d) $\sqrt{(\alpha^2 + \beta^2)}$

5. The line 3x + y – 9 = 0 divides the line joining the points (1, 3) and (2, 7) internally in the ratio

(a) 3 : 4 (b) 3 : 2 (c) 2 : 3 (d) 4 : 3 **

6. The ratio of the distances of point P(3, 4) from origin to that from y-axis is

(a) 3:5 (b) 5:3 (c) 5:4 (d) 3:4

MCQ Answer key:

1.(b) 2. (b) 3.(d) 4.(d) 5.(a) 6.(c)

Q. Find distance between A (10 cos θ, 0) and B (0, 10 sin θ). *

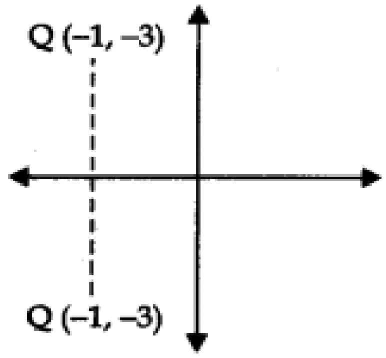
Answer:

$$AB = \sqrt{(0-10\cos\theta)^2 + (10\sin\theta - 0)^2}$$
$$= \sqrt{100\cos^2\theta + 100\sin^2\theta}$$
$$= \sqrt{100(\cos^2\theta + \sin^2\theta)}$$
$$= \sqrt{100} = 10 \text{ units}$$

Q**. Find the coordinates of reflection of Q (- 1, -3) in x-axis.**

Answer:

Reflection of Q(- 1, – 3) is (- 1, 3)

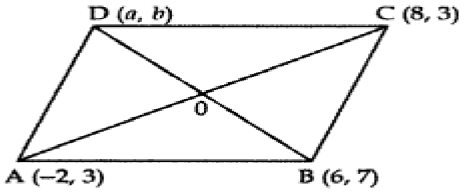


Q. Find the fourth vertex of parallelogram ABCD whose three vertices are A(- 2, 3), B(6, 7) and C(8, 3).

Answer:

Diagonals of parallelogram bisect each other,

mid-point of AC = mid-point of BD***



$$\left(\frac{6+a}{2}, \frac{7+b}{2}\right) = \left(\frac{-2+8}{2}, \frac{3+3}{2}\right)$$

$$\Rightarrow \left(\frac{6+a}{2}, \frac{7+b}{2}\right) = (3, 3)$$

$$\Rightarrow \frac{6+a}{2} = 3, \qquad \frac{7+b}{2} = 3$$

$$a = 0, \qquad b = -1$$

$$\therefore \quad D(0, -1)$$

Lets practice

Very short answer type questions

1. If the point (0, 2) is equidistant from the points (3, k) and (k, 5), find the value of k.
2. Find the ratio in which y-axis divides the line segment joining the points A(5, -6) and B (-1, -4). Also find the coordinates of the point of division.
3. Let P and Q be the points of trisection of the line segment joining the points A(2, -2) and B(-7, 4) such that P is nearer to A. Find the coordinates of P and Q.
4. Find the ratio in which P(4, m) divides the line segment joining the points A(2, 3) and B(6, -3). Hence find m. *
5. Point A(- 1, y) and B(5, 7) lie on a circle with centre 0(2, -3y). Find the values of y. Hence find the radius of the circle. **
6. The x-coordinate of a point P is twice its y-coordinate. If P is equidistant from Q(2, – 5) and R(-3,6), find the coordinates of P.*

Short answer type questions

7. AOBC is a rectangle whose three vertices are A(0, 4), O(0, 0) and B(3, 0). Find the length of its diagonal .

8.Prove that the points (3, 0), (6, 4) and (-1, 3) are the vertices of a right angled isosceles triangle. ***

9. If (1,p/3) is the mid point of the line segment joining the points (2, 0) and (0,2/9), then show that the line 5x + 3y + 2 = 0 passes through the point (- 1, 3p). ***

10. The vertices of a triangle are (- 2, 0), (2, 3) and (1, – 3). Is the triangle equilateral, isosceles or scalene? ***

11. If the point P(x, y) is equidistant from the points A(a + b, b – a) and B(a – b, a + b). Prove that bx = ay.

Long answer type questions

12. Show that ΔABC, where A(-2, 0), B(2, 0), C(0, 2) and Δ PQR where P(- 4, 0), Q(4, 0), R (0, 4) are similar triangles. ***

13. if A(-2, 1), B(a, 0), C(4, b) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of a and b. Hence find the lengths of its sides.

Case study based questions

Q. **Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. (assume that all distances covered are in straight lines and co-ordinates are in km).**

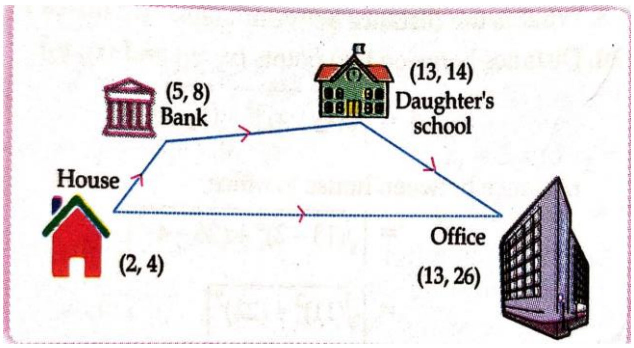
Answer the following questions:

14.what is the distance between house and bank?

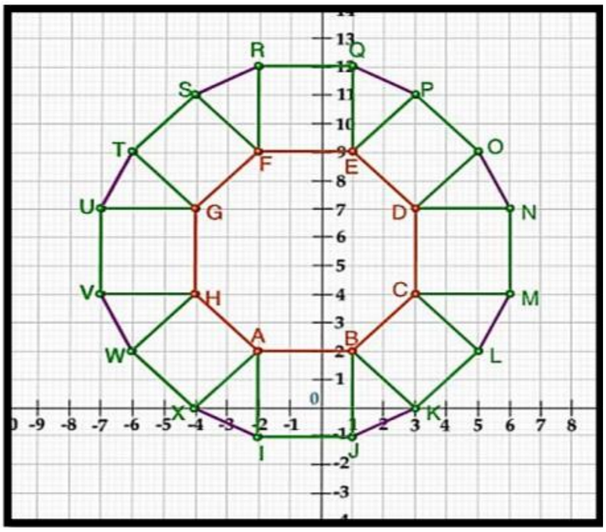
15.what is the distance between the bank and daughter's school?

16. What is the total distance travelled by ayush to reach the office?

17.what is the distance between house and office?



Q. A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find tessellation patterns on floors, walls, paintings etc. Shown below is a tiled floor in the archaeological Museum of Seville, made using squares, triangles and hexagons.



A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the Cartesian plane. He used regular octagons, squares and triangles for his floor tessellation pattern. Use the above figure to answer the questions that follow:

18. What is the length of the line segment joining points B and F?

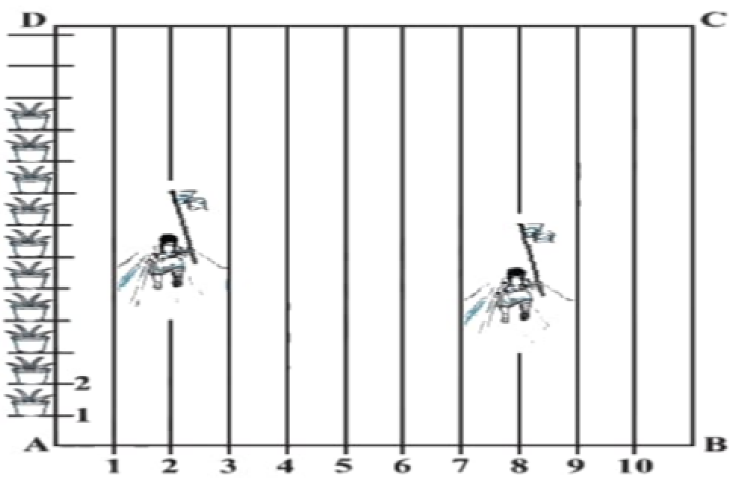
19. The centre 'Z' of the figure will be the point of intersection of the diagonals of quadrilateral WXOP. Then what are the coordinates of Z?

20. What are the coordinates of the point on y axis equidistant from A and G?

OR

What is the area of area of Trapezium AFGH

Q. To conduct Sport Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1m each. 80 flower pots have been placed at a distance of 1 m from each other along AD, as shown in figure Hannah runs 1/4 th the distance AD in the 2nd line and posts a blue flag. Preeta runs 1/5 th the distance AD on the 8th line and posts a green flag.



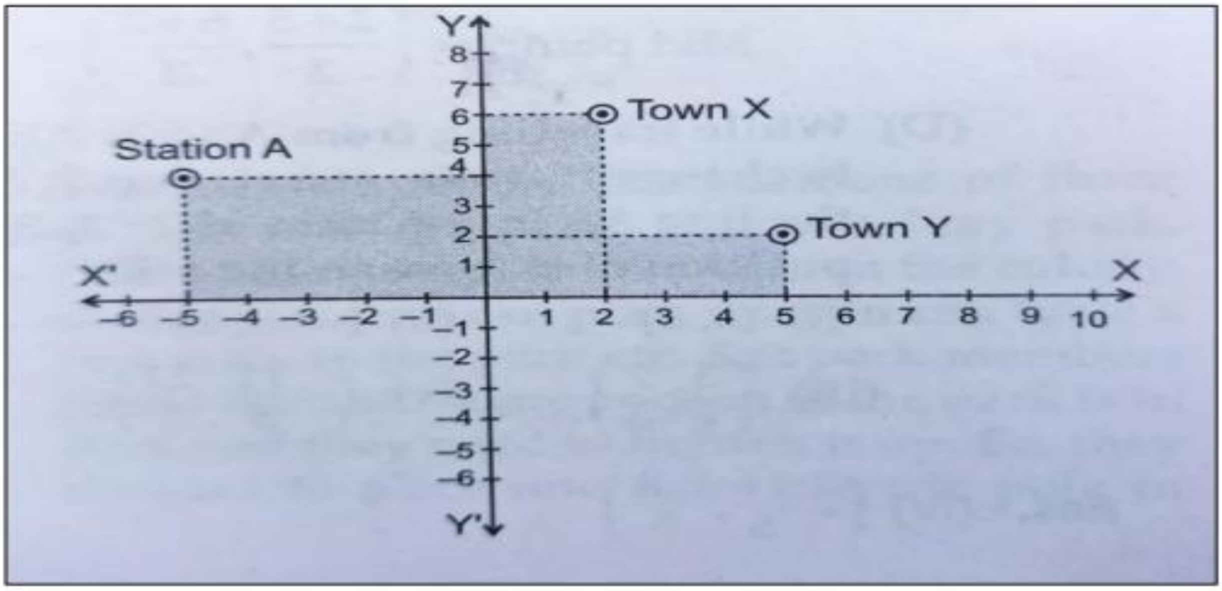
21. What is the distance between both the flags?

22. If Uthara has to post an orange flag exactly halfway between the line segment joining the two flags, where should she post her flag?

23. Which mathematical concept is used in the above problem?

24. What value is depicted in this problem?

25. Two friends Dalvin and Alice works in the same office in Toronto. In the Christmas vacation, they both decided to go to their home towns represented by Town X and Town Y. Town X and Town Y are connected by trains from the same station C in Toronto. The situation of Town X, Town Y and station A is shown on the coordinate axis.



Based on the given situation, answer the following questions:

i. What is the distance that Dalvin have to travel to reach his hometown X ?

(a) √51 units (b) √53 units (c) √35 units (d) √47 units

ii. What is the distance that Alice has to travel to reach her hometown Y?

(a) 2√26 units (b) √107 units (c) 2√10 units (d) √51 units

iii. Now, both of them plan to meet at a place between Town X and Town Y, such that it is a mid-point between both. Calculate the coordinates of the mid-point of X and Y.

(a) (1, 3) (b) (2, - 4) (c) (2.5,3) (d) (3.5, 4)

iv. While travelling from A to Y, Alice had to change the train, at a station, it divides the line AY in the ratio of 2: 3, find the position of station on the grid.

(a) (0, 7 9) (b) (− 11 5 , 24 5) (c) (11 8 , 17 3) (d) (12, 7)

CHAPTER- 7 Answer key

1. k = 1. 2 . -13/3 3. (-1,0) and (-4,2) 4. m=0

5. 5 units 6. (16,8) 7. 5 units 10. scalene triangle

13..a=1,b=1; length of each side √10 units

14. 5km 15. 10km 16. 27km 17. 24.6km

18.√58 units 19.(-½ ,11/2) 20. (0, 5.7) or 10 square units

21. √61m 22. Co-ordinate Geometry 23. (5, 45 2) 24. Team Spirit

25 . (i) (b) √53 units (ii) (a) 2√26 units (iii) (d) (3.5, 4) (iv) (b) (− 11/5 , 24/5)

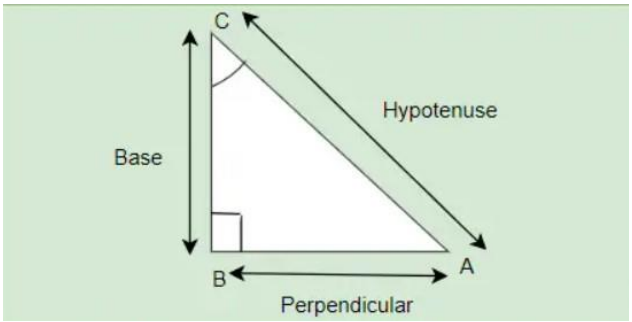
CHAPTER - 8

INTRODUCTION TO TRIGONOMETRY

Prepared by- DEEPAK SINGH BORA
TGT (MATHS)
KV SINGHARSI

TRIGONOMETRY: A branch of mathematics that deals with the relationships between the angles and sides of triangles. It has numerous applications in various fields, including physics, engineering, and architecture.

TRIGONOMETRIC RATIOS: The ratios of the sides of a right triangle with respect to its acute angles, called trigonometric ratios of the angle.



For acute angle C, side opposite to C is perpendicular, adjacent side BC is base and side opposite to angle B is hypotenuse

so Trigonometric ratios of an acute angle C in a right angle triangle are as follows

FOR ANGLE C	FOR ANGLE A
sin C=Perpendicular/Hypotenuse=AB/AC cosec C= Hypotenuse/Perpendicular=AC/AB cos C=Base/Hypotenuse=BC/AC sec C=Hypotenuse/Base=AC/BC tan C=Perpendicular/Base=AB/BC cot C=Base/Perpendicular=BC/AB	sin A=Perpendicular/Hypotenuse=BC/AC cosec A= Hypotenuse/Perpendicular=AC/BC cos A=Base/Hypotenuse=AB/AC sec A=Hypotenuse/Base=AC/AB tan A=Perpendicular/Base=BC/AB cot A=Base/Perpendicular=AB/BC

EXAMPLE 1: In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. Determine:

(i) sin A, cos A

(ii) sin C, cos C

Solution:

In a given triangle ABC, right angled at B = ∠B = 90°

Given: AB = 24 cm and BC = 7 cm

According to the Pythagoras Theorem,

In a right- angled triangle, the squares of the hypotenuse side is equal to the sum of the squares of the other two sides.

By applying Pythagoras theorem, we get

$AC^2=AB^2+BC^2$

$AC^2 = (24)^2+7^2$

$AC^2 = (576+49)$

$AC^2 = 625cm^2$

$AC = \sqrt{625} = 25$

Therefore, AC = 25 cm

(i) To find Sin A, Cos A

We know that sine (or) Sin function is the equal to the ratio of length of the opposite side to the hypotenuse side. So it becomes

Sin A = Perpendicular /Hypotenuse = BC/AC = 7/25

Cosine or Cos function is equal to the ratio of the length of the adjacent side to the hypotenuse side and it becomes,

Cos A = Base /Hypotenuse = AB/AC = 24/25

Relation between Trigonometric Ratios:

- **Cosec A=1/sinA**
- **secA = 1/cosA**
- **tan A = sinA/cosA**
- **cotA = cosA/sinA=1/tanA**

EXAMPLE 2: Given sec θ = 13/12 Calculate all other trigonometric ratios

Solution:

We know that sec function is the reciprocal of the cos function which is equal to the ratio of the length of the hypotenuse side to the adjacent side

Let us assume a right angled triangle ABC, right angled at B

sec θ =13/12 = Hypotenuse/Adjacent side = AC/AB

Let AC be 13k and AB will be 12k

Where, k is a positive real number.

According to the Pythagoras theorem, the squares of the hypotenuse side is equal to the sum of the squares of the other two sides of a right angle triangle and we get,

$AC^2 = AB^2 + BC^2$

Substitute the value of AB and AC

$(13k)^2 = (12k)^2 + BC^2$

$169k^2 = 144k^2 + BC^2$

$169k^2 = 144k^2 + BC^2$

$BC^2 = 169k^2 - 144k^2$

$BC^2 = 25k^2$

Therefore, BC = 5k

Now, substitute the corresponding values in all other trigonometric ratios

So,

Sin θ = Opposite Side/Hypotenuse = BC/AC = 5/13

Cos θ = Adjacent Side/Hypotenuse = AB/AC = 12/13

tan θ = Opposite Side/Adjacent Side = BC/AB = 5/12

Cosec θ = Hypotenuse/Opposite Side = AC/BC = 13/5

cot θ = Adjacent Side/Opposite Side = AB/BC = 12/5

TRIGONOMETRIC RATIOS OF SOME SPECIFIC ANGLES:

θ	0^0	30^0	45^0	60^0	90^0
sinθ	**0**	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	**1**
cosθ	**1**	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	**0**
tanθ	**0**	$\frac{1}{\sqrt{3}}$	**1**	$\sqrt{3}$	**Not defined**
cotθ	**Not defined**	$\sqrt{3}$	**1**	$\frac{1}{\sqrt{3}}$	**0**
secθ	**1**	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	**2**	**Not defined**
cosecθ	**Not defined**	**2**	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	**1**

Example 3:

Evaluate: sin 60° cos 30° + sin 30° cos 60°

Solution:

sin 60° cos 30° + sin 30° cos 60°

First, find the values of the given trigonometric ratios

sin 30° = 1/2

cos 30° = √3/2

sin 60° = √3/2

cos 60°= 1/2

Now, substitute the values in the given problem

sin 60° cos 30° + sin 30° cos 60° = √3/2 ×√3/2 + (1/2) ×(1/2) = 3/4+1/4 = 4/4 =1

Example 4:

Evaluate: $2tan^2 45^0 + cos^2 30^0 - sin^2 60^0$

Solution: As $\tan 45^0=1$, $\cos 30^0=\frac{\sqrt{3}}{2}$, $\sin 60^0=\frac{\sqrt{3}}{2}$

Therefore putting the values in given expression;

$=2(1)^2+(\frac{\sqrt{3}}{2})^2-(\frac{\sqrt{3}}{2})^2$

= 2+0=2

Trigonometric Identities: An equation involving trigonometric ratios of an angle is called a trigonometric identity if it is true for all values of the angle:

- **$\sin^2 A+\cos^2 A=1$** or $\sin^2 A=1- \cos^2 A$ or $\cos^2 A=1- \sin^2 A$
- **$1+\tan^2 A=\sec^2 A$** or $\sec^2 A- \tan^2 A =1$ or $\tan^2 A = \sec^2 A -1$
- **$1+\cot^2 A=\mathrm{cosec}^2 A$** or $\mathrm{cosec}^2 A- \cot^2 A=1$ or $\cot^2 A= \mathrm{cosec}^2 A-1$

EXAMPLE 5: **EVALUATE: 9 $\sec^2 A$ – 9 $\tan^2 A$**

SOLUTION:Take 9 outside, and it becomes

9 $\sec^2 A$ – 9 $\tan^2 A$

= 9 ($\sec^2 A$ – $\tan^2 A$)

= 9×1 = 9 (∵ $Sec^2 A - tan^2 A$ = 1)

Therefore, $9 \sec^2 A - 9 \tan^2 A = 9$

EXAMPLE 6:

Prove that: $(\text{cosec } \theta - \cot \theta)^2 = (1-\cos \theta)/(1+\cos \theta)$

Proof: To prove this, first take the Left-Hand side (L.H.S) of the given equation, to prove the Right Hand Side (R.H.S)

L.H.S. = $(\text{cosec } \theta - \cot \theta)^2$

The above equation is in the form of $(a-b)^2$, and expand it

Since $(a-b)^2 = a^2 + b^2 - 2ab$

Here $a = \text{cosec } \theta$ and $b = \cot \theta$

$= (\text{cosec}^2\theta + \cot^2\theta - 2\text{cosec } \theta \cot \theta)$

Now, apply the corresponding inverse functions and equivalent ratios to simplify

$= (1/\sin^2\theta + \cos^2\theta/\sin^2\theta - 2\cos \theta/\sin^2\theta)$

$= (1 + \cos^2\theta - 2\cos \theta)/(1 - \cos^2\theta)$

$= (1-\cos \theta)^2/(1 - \cos\theta)(1+\cos \theta)$

$= (1-\cos \theta)/(1+\cos \theta)$ = R.H.S.

Therefore, $(\text{cosec } \theta - \cot \theta)^2 = (1-\cos \theta)/(1+\cos \theta)$

Hence proved.

PRACTICE QUESTIONS SECTION-A

MULTIPLE CHOICE QUESTIONS:

1. sin 2B = 2 sin B is true when B is equal to

(a) 90° (b) 60° (c) 30° (d) 0°

*2.The value of cos 0°. cos 1°. cos 2°. cos 3°… cos 89° cos 90° is

(a) 1 (b) -1 (c) 0 (d) $\frac{1}{\sqrt{2}}$

3.What is the minimum value of sin A, $0 \leq A \leq 90°$

(a) -1 (b) 0 (c) 1 (d) 1/2

4. If θ is an acute angle of a right angled triangle ,then which of the following equation is not true?

(a) $sin\theta cot\theta = \cos\theta$ (b) $cos\theta tan\theta = sin\theta$

(c) $cosec^2\theta - cot^2\theta = 1$ (d) $tan^2\theta - sec^2\theta = 1$

*5. If $\sin \theta + \sin^2 \theta = 1$, then $\cos^2 \theta + \cos^4 \theta =$

(a) -1 (b) 0 (c) 1 (d) 2

**6. 5 $\tan^2 A - 5 \sec^2 A + 1$ is equal to

(a) 6 (b) -5 (c) 1 (d) -4

7. If sin A = 1/2 and cos B = 1/2, then A + B = ?

(a) 90^0 (b) 30^0 (c) 60^0 (d) 0^0

**8. If 4tanA=3, then $\frac{4sinA-cosA}{4sinA+cosA}$

(a) 2/3 (b) 1/3 (c) 1/2 (d) ¾

9.(sin30° + cos30°) – (sin 60° + cos60°)

(a) – 1 (b) 0 (c) 1 (d) 2

10. Value of tan30°/cot60° is:

(a) 1/√2 (b) 1/√3 (c) 1 (d) √3

**11. If x tan 45° sin 30° = cos 30° tan 30°, then x is equal to

(a) √3 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

***12. If sec A + tan A = x, then tan A =

(a) $\frac{x^2-1}{x}$ (b) $\frac{x^2-1}{2x}$ (c) $\frac{x^2+1}{x}$ (d) $\frac{x^2+1}{2x}$

13. $\frac{1-cosA}{sinA}$ is equal to

(a) $\frac{sinA}{1-cosA}$ (b) $\frac{sinA}{1+cosA}$ (c) $\frac{cosA}{1-cosA}$ (d) $\frac{cosA}{1+cosA}$

**14. If sin A – cos A = 0, then the value of $\sin^4 A + \cos^4 A$ is

(a) 2 (b) 1 (c) $\frac{3}{4}$ (d) $\frac{1}{2}$

15. If in ΔABC, ∠C = 90°, then sin (A + B) =

(a) 0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 1

**16. 9 $\sec^2 A - 9 \tan^2 A$ =

(a) 1 (b) 9 (c) 8 (d) 0

17. (1 + tan θ + sec θ) (1 + cot θ – cosec θ)

(a) 0 (b) 1 (c) 2 (d) – 1

18. (sec A + tan A) (1 – sin A) =

(a) sec A (b) sin A (c) cosec A (d) cos A

*19. $1+\tan^2 A/1+\cot^2 A$ =

(a) $\sec^2 A$ (b) -1 (c) $\cot^2 A$ (d) $\tan^2 A$

**20.2tan 30°/1+tan^{2}30° =

(a) sin 60° (b) cos 60° (c) tan 60° (d) sin 30°

SECTION-B

**21.Evaluate: $5cosec^2 45^0 - 3sin^2 90^0 + 5cos0^0$

22.If tan$\theta = \frac{3}{4}$, then find the value of $cos^2\theta - sin^2\theta$.

23.Find the value of $\frac{cos30^0 + sin60^0}{1 + cos60^0 + sin30^0}$.

24.Given 15cotA=8,findsinA and secA.

*25.Evaluate: $sin60^0 cos30^0 + sin30^0 cos60^0$

**26.If cot$\theta = \frac{7}{8}$,Evaluate: $\frac{(1+sin\theta)(1-sin\theta)}{(1+cos\theta)(1-cos\theta)}$.

*27. If sinx+cosy =1,and x=30^0 and y is an acute angle, find the value of y.

28. If sin$\theta = x \ and \ sec\theta = y \ , find \ cot\theta$.

29. Express cosA in terms of cotA.

**30. If sinA=cosA, find $2tan^2A + sin^2A - 1$.

*31. If $\sqrt{3}sin\theta - cos\theta = 0 \ and \ 0 < \theta < 90, find \ the \ value \ of \ \theta$.

***32. If $sec\theta + tan\theta = p$ then find the value of cosecθ.

33.In $\Delta ABC \ right \ angled \ at \ C \ and \ AC = \sqrt{3}$BC, then prove that: $\measuredangle ABC = 60^0$.

*34. If (1+cosA)(1-cosA)=3/4,find secA.

*35. If cosecA=5/3,Find cosA+tanA

SECTION-C

***36 .Prove that:$\sqrt{sec^2\theta + cosec^2\theta}$ = tan$\theta + cot\theta$

**37. Prove that $\frac{cotA-cosA}{cotA+cosA} = \frac{cos^2A}{(1+sinA)^2}$

38. Prove that (sec$\theta + tan\theta)(1 - sin\theta) = cos\theta$

*39. Prove that $\frac{1+secA}{secA} = \frac{sin^2A}{1-cosA}$

**40. If cos$\theta + sin\theta = \sqrt{2}cos\theta, show\ that\ cos\theta - sin\theta = \sqrt{2}\,sin\theta$

41. Prove that $\frac{1+cotA-cosecA}{1+tanA+secA} = 2$

*42. Prove that $\frac{tan\theta}{1-tan\theta} - \frac{cot\theta}{1-cot\theta} = \frac{cos\theta+sin\theta}{cos\theta-sin\theta}$

43. If sin$\theta = \frac{12}{13}$,Find $\frac{sin^2\theta-cos^2\theta}{2sin\theta cos\theta} \times \frac{1}{tan^2\theta}$

***44. Prove that $\frac{sin^3\theta+cos^3\theta}{sin\theta+cos\theta} = 1 - sin\theta cos\theta$

**45. Prove that $\sqrt{\frac{1+cos\theta}{1-cos\theta}} = cosec\theta + cot\theta$

SECTION-D

*46. If 7sin^{2}A+3cos^{2}A=4,then show that tanA= $\frac{1}{\sqrt{3}}$

*47. Evaluate:$\frac{4cot^2 60^0+sec^2 30^0-2sin^2 45^0}{sin^2 60^0+cos^2 45^0}$

**48. If sin θ + cos θ = √3, then prove that tan θ + cot θ = 1.

**49. Prove that:$\frac{tan\theta}{1+cot\theta} + \frac{cot\theta}{1-tan\theta} = 1 + sec\theta cosec\theta$

**50. Prove that:$\sqrt{\frac{sec\theta-1}{sec\theta+1}} + \sqrt{\frac{sec\theta+1}{sec\theta-1}} = 2cosec\theta$

**51. Prove that:(sin θ + sec θ)^2 + (cos θ + cosec θ)^2 = (1 + sec θ cosec θ)^2

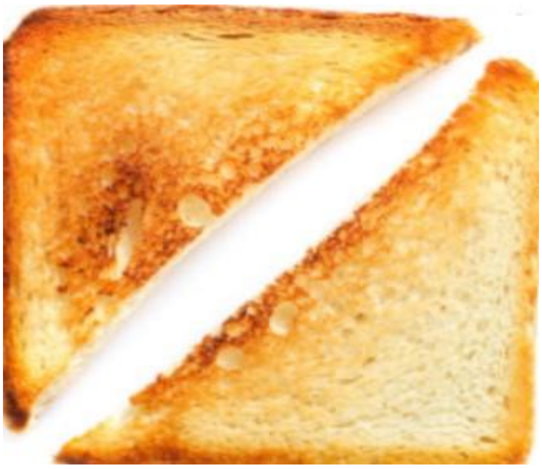
*52. Prove that: 3 (sin θ – cos θ)^4 + 6 (sin θ + cos θ)^2 + 4 (sin^6 θ + cos^6 θ) = 13.

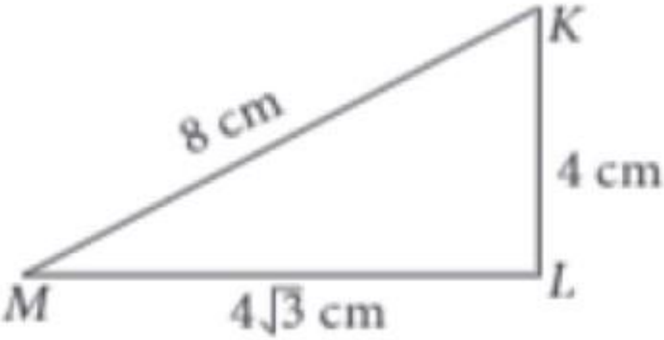
*53. Prove that:sin A(1 + tan A) + cos A(1 + cot A) = sec A + cosec A

***54. If sec θ – tan θ = x, show that: sec θ=1/2(x+1/x) and tan θ=1/2(1/x-x)

CASE STUDY QUESTIONS

55. Astha is feeling so hungry and she thought to eat something. She looked into the fridge and found a bread pieces. She decided to make a sandwich. She cut the piece of bread diagonally and found it forms a right-angled triangle, with sides 4 cm, 4√3 cm and 8 cm.





On the basis of above information, answer the following questions.

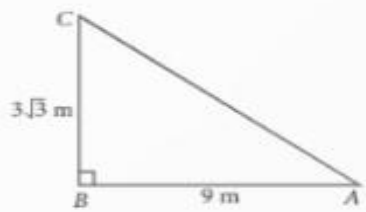
(i) The value of ∠M is : A. 30° B. 60° C. 45° D. None of these

(ii) The value of ∠K is : A. 45° B. 30° C. 60° D. None of these

(iii) Find the value of tan$M$: A. √3 B. 1/√3 C. 1 D. None of these

(iv)$\sec^2 M - 1 =$?: A. tanM B. $tan^2 M$ C. tan2M D. None of these

56. Three friends - Om, Nitish and Sahil are playing hide and seek in a park. Om and Nitish hide in the shrubs and Sahil have to find both of them. If the positions of three friends are at A, Band C respectively as shown in the figure and forms a right angled triangle such that AB = 9 m, BC = 3√3 m and ∠B = 90°, then answer the following questions.



(i) The measure of $\angle$A is :

(a) 30^0 (b) 45^0 (c) 60^0 (d) none of these

(ii) The measure of $\angle$C is :

(a) 30^0 (b) 45^0 (c) 60^0 (d)none of these

(iii) The length of AC is:

(a) $\sqrt{3}$ m (b) $2\sqrt{3}\ m$ (c) $4\sqrt{3}$ m (d) $6\sqrt{3}$ m

(iv) cos2A=

(a)0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d)$\frac{\sqrt{3}}{2}$

(v) sin$\frac{C}{2}$ =

(a)0 (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d)$\frac{\sqrt{3}}{2}$

Answer key CHAPTER-8 SOLUTIONS:

1.d , 2.c , 3.b , 4.d , 5.d , 6.d ,7.a ,8.c ,9.b ,10.c ,11.d ,12.b , 13.b , 14.d ,15.d ,16.b ,17.c 18.d ,19.d ,20.a , 21.12 , 22.7/25 ,23.$\frac{\sqrt{3}}{2}$,24. sinA=15/17,secA=17/8 ,25.1 ,26. 49/64 ,27.y = 60^0 ,28. 1/xy ,29.$cosA$=$\frac{cotA}{\sqrt{1+cot^2A}}$,30.3/2 ,31.30^0 ,32.$cosec\theta = \frac{p^2+1}{p^2-1}$,34. 2 ,35.31/20 ,43. 595/3456 , 47. 4/3 ,55.(i) a (ii)c (iii)b (iv) c ,56.(i)a (ii)c (iii)d (iv)b (v)d

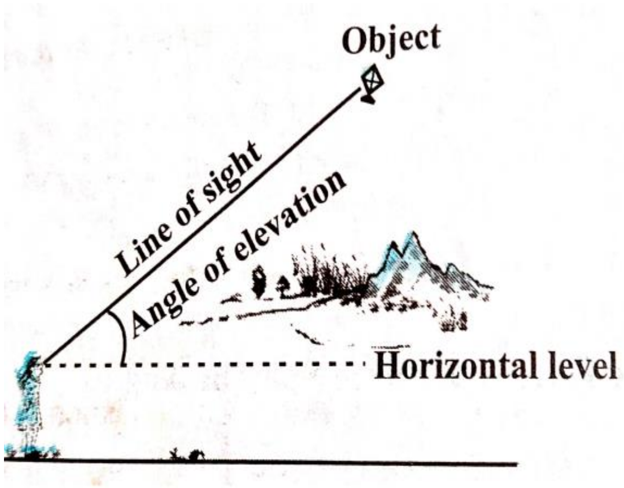
CHAPTER -9

SOME APPLICATION OF TRIGONOMETRY

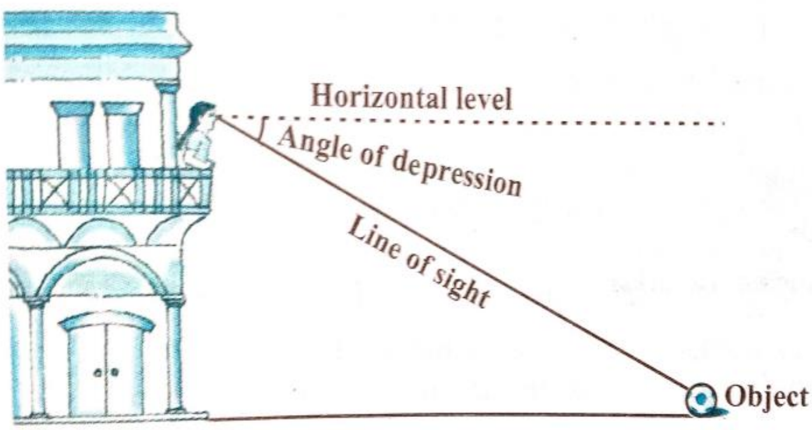
Prepared by- A K MAHTO
TGT (MATHS)
KV HINOO 2nd SHIFT

BASIC CONCEPT :

ANGLE OF ELEVATION : It is the angle formed by the line of sight with the horizontal through the eyes of observer when the object is above the horizontal level.



ANGLE OF DEPRESSION : It is the angle formed by the line of sight with the horizontal when the object is below the horizontal level.



SECTION - A MCQ

Q.1. If the height of a vertical pole is equal to the length of its shadow on the ground ,the angle of elevation of the sun is

a) 0° b) 30° c) 45° d) 60°

Ans - (c)

Q.2. If the length of the shadow of a tower is √3 times its height then the angle of elevation of the sun is

a) 45° b) 30° c) 60° d) 90°

Ans - (b)

Q.3. A ladder makes an angle of 60° with the ground when placed against a wall. If the foot of the ladder is 2 m away from the wall , the length of the ladder is

a) 4/√3 b) 4√3 c) 2√2 d) 4 m

Ans – (d)

Q.4. The angle of depression of a car parked on the road from the top of a 150 m high tower is 30°. The distance of the car from the tower is

a) 50√3 b) 150√3 c) 150√2 d) 75

Ans - (b)

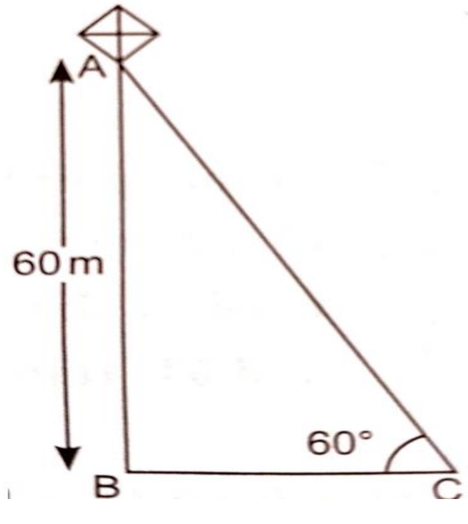
Q.5. The angle of elevation of the top of a tower from a point on the ground 30 m away from the foot of the tower is 30° .The height of the tower is

a) 30 m b) 10√3 m c) 20 m d) 10√2 m

Ans - (b)

SECTION - B SHORT ANSWER (Type – II)

Q.1. A kite is flying at a height of 60 m above the ground . The string attached to the kite is temporarily tied to a point on the ground . The inclination of the string with the ground is 60°. Find the length of the string.



Solution - The height of the kite is 60 m.

In ΔABC , AB/AC = sin60°

AC = 40√3 m.

Q.2. If at some time of a day the ratio of the height of a vertically standing pole to the length of its shadow on the ground is √3:1 then find the angle of elevation of the at the time.

Solution : In right triangle BAC,

Tan Θ = AB/AC =$\sqrt{3}$x/x

So, $\Theta = 60°$.

Q.3. The angle of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Show that the height of the tower is 6 m.

Q.4. From the top of a building AB, 60 m high , the angle of depression of the top and bottom of a vertical lamp post CD are observed to be 30° and 60° respectively.

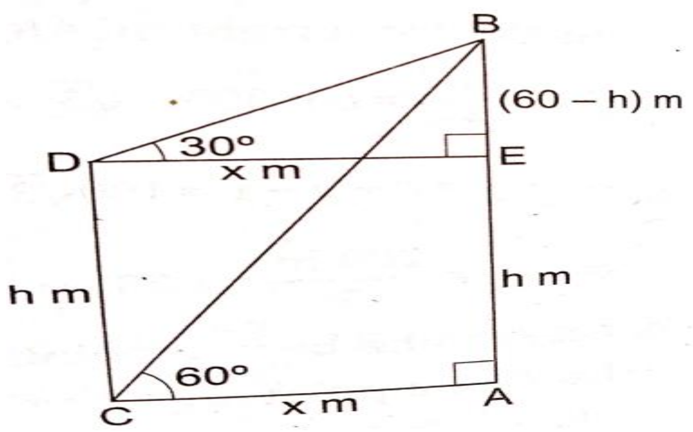
Find

i) the horizontal distance between AB and CD ,

ii) the height of the lamp post,

iii) the difference between the heights of the building and the lamp post.

Ans - i) 34.64 ii) 40 m iii) 20 m

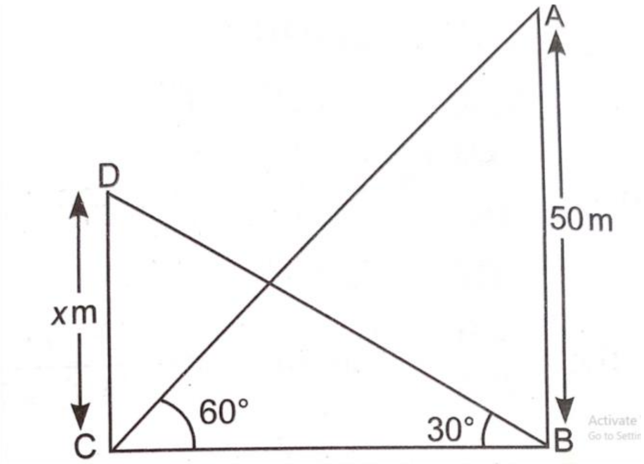


Q.5. An aeroplane is flying at a height of 300m above the ground . Flying at this height , the angle of depression from the aeroplane of two points on both banks of a river in opposite direction are 45° and 60° respectively. Find the width of the river.

Ans – 473.2 m.

**** Q.1. Case Study – 1**

The angle of elevation of the top of a building from the foot of a tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° .The height of the tower is 50 m . Observe the given figure and give the answer of the following :.



(i) What is the distance between building CD and tower AB ? (1)

(ii) What is the distance between root of building CD and the top of the tower AB ? (1)

(iii) Find the height of building CD ? (2)

OR

Find the distance between root of tower AB and the top of the building CD ?

SOLUTION :

(i) In Right triangle ABC, tan 60° = AB/BC , So BC=50/√3

(ii) In Right triangle ABC , Sin60° = 50/AC So, AC =100/√3

(iii In Right triangle DCB, tan30° =DC/BC , CD= x = 50/3

OR,

In Right triangle DCB, sin30° = CD/BD So, BD=100/3

*****Q.2.. Case study-2**

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground . The angle of elevation of the balloon from the eyes of the girl at any instant is 60° . after some time the angle of elevation reduces to 30° .

Use the above information give the answer of the following:

i) Draw a neat labelled diagram to show the above situation diagrammatically. (1)

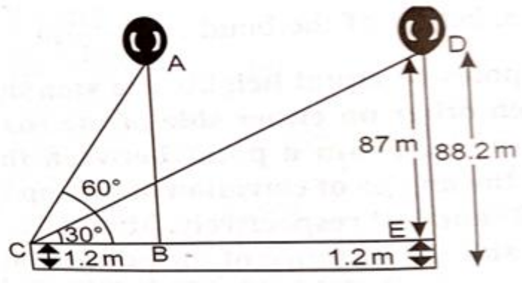
ii) Find the vertical height of the balloon from the eyes of girl. (1)

iii) Find the distance travelled by the balloon after some time when angle of elevation reduces to 30°. (2)

OR

Find the distance travelled by the balloon after some time when angle of elevation reduces to 45°.

SOLUTION :(i)



(ii) Vertical height of the balloon from the eyes of girl= 88.2-1.2=87m

(iii) In Δ DEC tan30° = DE/CE

CE = 87√3 m

Or

In Δ DEC tan45° = DE/CE

CE = 87 m

**** Q.3.CASE STUDY -3**

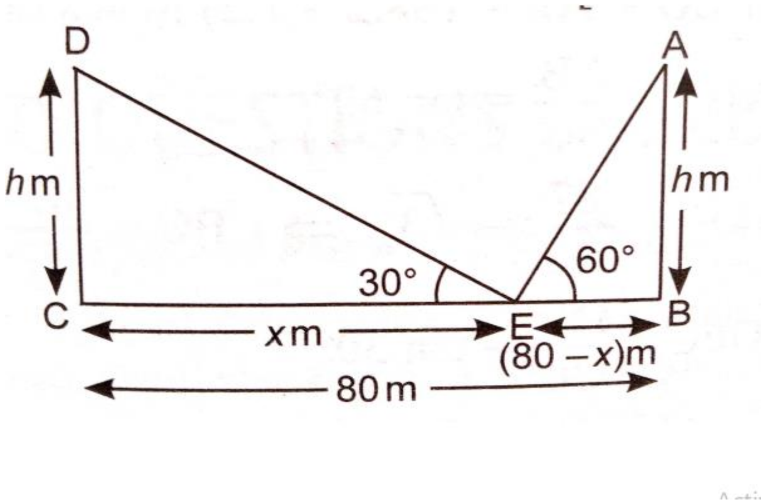
Two poles of equal heights are standing opposite to each other on either side of the road which is 80 m wide. From a point in between them on the road, the anglesof elevation of the top of poles are 60^0 and 30^0 respectively. Find the height of the poles and the distances of the point from the poles.

(i) Draw a neat labelled diagram to show the above situation diagrammatically. (1)

(ii) Find the height of the pole? (2)

(iii) Find the distance of the point from the pole? (1)

SOLUTION : (i)



(ii) In $\Delta\ CDE$, tan 30° = CD/CE

X = √3h -----(i)

In $\Delta\ ABE$, tan 60° = AB/BE

h/(80-x) = √3

h= 80√3 - √3x -------(ii)

Solving (i) and (ii), h=20√3 m.

(iii)x= √3h = √3 x 20√3 =60 m

And 80-x = 80- 60 = 20 m.

Q.4. CASE STUDY - 4

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60° .The height of the tower is 50m .

Read the above information carefully and give the answer of the following:

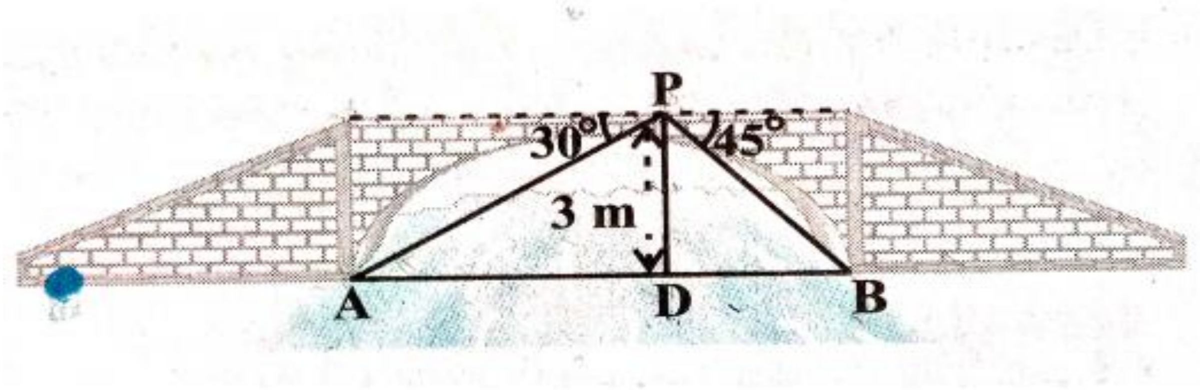
(i) Draw the neat labelled diagram for the given situation. (1)

(ii) Find the distance between tower and building. (Hint - Distance = 50/√3) (2)

(iii) Find the height of the building? (Hint- Height of building= 50/3) (1)

*** Q.5. CASE STUDY - 5

From a point on a bridge across a river, the angles of depression of the banks on opposite sides of the river are 30° and 45°, respectively. The height of the bridge is 3 m from the banks of the river.



Read the above situation and observe the above diagram, then give the answer :

(i) Find the length between edge A and point D. (Hint- AD= $3\sqrt{3}$ m) (1)

(ii) Find the length between edge B and point D. (Hint – BD = 3m) (1)

(iii) Find the width of the river AB. (Hint – AB= $3(\sqrt{3}+1)$) (2)

Q.6. CASE STUDY - 6

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. Since one ship is exactly behind the other on the same side of the lighthouse.

Read the above situation carefully and give the answer of the following :

(i) Draw the labelled diagram of the given situation. (1)

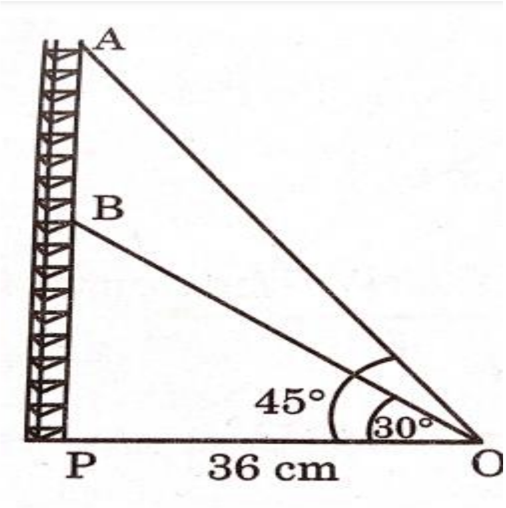
(ii) Find the distance between first ship and lighthouse. (Hint- Distance= 75 m) (1)

(iii) Find the distance between two ships. (Hint- Distance = 54.75) (2)

** Q.7. CASE STUDY -7

Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept. A radio station tower was built in two sections A and B. Tower is supported by wires from a point O.

Distance between the base of the tower and point O is 36 m. from point O , the angle of elevation of the top of the section B is 30° and the angle of elevation of the top of Section is 45° .



Read the above information and solve the following problems :

(i) Find the length of the wire from the point O to the top of section B. (Hint- 24√3 m) (1)

(ii) Find the distance AB. (Hint- AB= 12 (3 -√3) cm (2)

OR

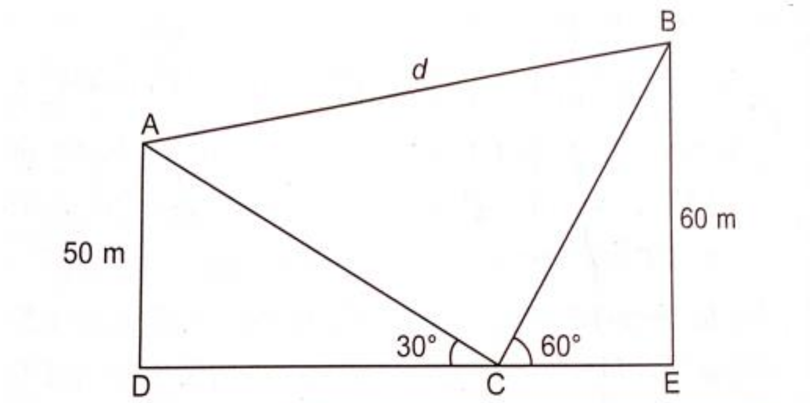
Find the area of Δ OPB . (Hint- 216 √3 cm^2)

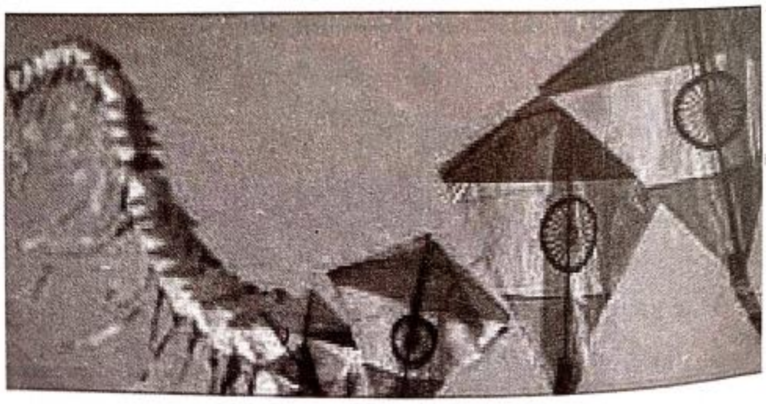
(iii) Find the height of Section A from the base of the tower. (Hint- AP= 36 cm) (1)

***Q.8. CASE STUDY -8 KITE FESTIVAL

Kite festival is celebrated in many countries at different times of the year. In India every year 14th January is celebrated as International Kite Day. On this day many people visit India and participate in the festival by flying various kinds of kites.

The picture given below,shows three kites flying together In the given figure ,the angle of elevation of two kites (point A and B) from the hands of a man (point C) are found to be 30° and 60° respectively, Taking AD = 50 m and BE= 60 m.





Give the answer of the following as per above information :

(i) Find the distance of CD. (Hint- $50\sqrt{3}$ m) (1)

(ii) What is the distance between the kite B and the point C. (Hint - $40\sqrt{3}$ m) (1)

(iii) Calculate the distance DE. (Hint- DE= CD + CE = $50\sqrt{3} + 20\sqrt{3} = 70\sqrt{3}$ m) (2)

******

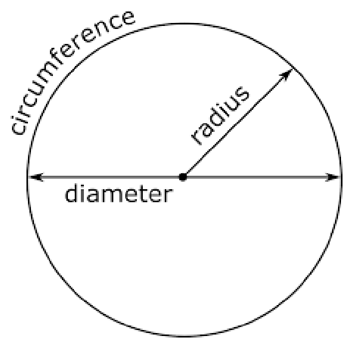
CHAPTER -10

CIRCLES

Prepared by- KASHINATH MAHAPATRA
TGT (MATHS)
KV CHAKRADHARPUR

Introduction to Circles

As we know that a circle is a closed two-dimensional geometrical figure, such that all points on the surface of a circle are equidistant from the point called the "**centre**". The distance from the centre to any point on the surface of a circle is called **"Radius**".



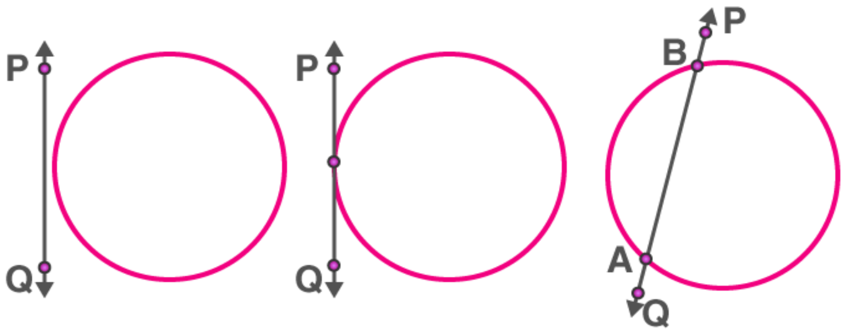
Circle and Line in a Plane

For a circle and a line on a plane, there can be **three** possibilities.

i) they can be **non-intersecting**

ii) they can have **a single common point:** in this case, the line touches the circle.

ii) they can have **two common points:** in this case, the line cuts the circle.

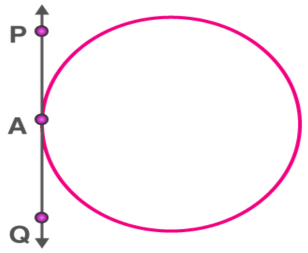


(i) Non-intersecting (ii) Touching (iii) Intersecting

Important Concepts

Tangent to a circle

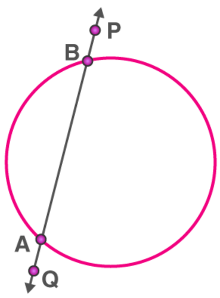
*A tangent to a circle is a line that intersects the circle at only one point.



* There is only one tangent at a point on a circle.
* There are exactly two tangents to a circle through a point lying outside the circle.
* The tangent at any point of a circle is perpendicular to the radius through the point of contact.
* The length of tangents drawn from an external point to a circle are equal.

Secant

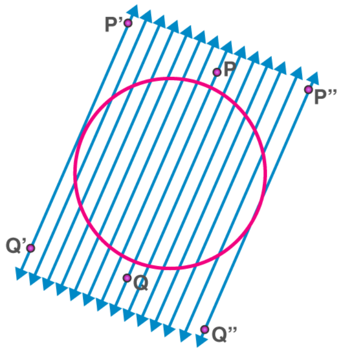
A **secant to a circle** is a line that has two points in common with the circle. It cuts the circle at two points, forming a chord of the circle.



Secant

Two Parallel Tangents at most for a Given Secant

For every given **secant** of a circle, there are **exactly two tangents which are parallel** to it and touches the circle at two **diametrically opposite points.**



Parallel tangents

From the given diagram, we can observe the following points:

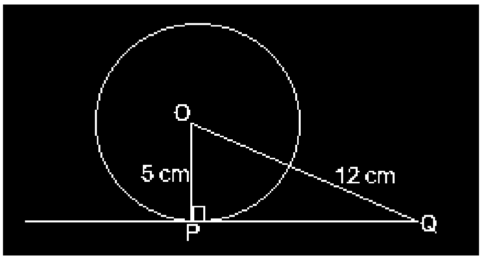
- PQ is the secant of a circle.
- P'Q' & P"Q" are two tangents which are parallel to PQ.

SOLVED MCQs (1 MARK)

1. A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm

Answer:



In the above figure, the line that is drawn from the centre of the given circle to the tangent PQ is perpendicular to PQ.

And so, $OP \perp PQ$

Using Pythagoras' theorem in triangle ΔOPQ, we get,

$OQ^2 = OP^2+PQ^2$

$(12)^2 = 5^2+PQ^2$

$PQ^2 = 144-25$

$PQ^2 = 119$

$PQ = \sqrt{119}$ cm

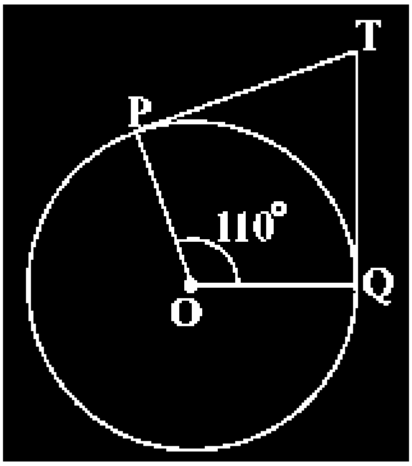
So, **option D,** i.e., $\sqrt{119}$ cm, is the length of PQ.

2. If TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to

(A) 60° (B) 70° (C) 80° (D) 90°

Answer:

From the question, it is clear that OP is the radius of the circle to the tangent PT, and OQ is the radius to the tangent TQ.



So, OP ⊥ PT and TQ ⊥ OQ

∴ ∠OPT = ∠OQT = 90°

Now, in the quadrilateral POQT, we know that the sum of the interior angles is 360°.

So, ∠PTQ+∠POQ+∠OPT+∠OQT = 360°

Now, by putting the respective values, we get

∠PTQ +90°+110°+90° = 360°

∠PTQ = 70°

So, ∠PTQ is 70° which is option B.

PRACTICE MCQ BASED QUESTIONS (1 MARK)

Q.1 The common point of a tangent to a circle with the circle is called ….***

(a) Centre (b) point of contact (c) end point (d) none of these.

Q.2 If radii of two concentric circles are 4 cm and 5 cm, then the length of each chord of one circle which is tangent to the other circle is ____***

(a) 3cm (b) 1cm (c) 6cm (d) 9cm

Q.3 If angle between two radii of a circle is 130º, the angle between the tangents at the ends of the radii is : ***

(a) 90º (b) 50º (c) 70º (d) 40

Q.4. A circle can have _____parallel tangents at a single time. ***

(a) One (b) Two (c) Three (d) Four

Q.5. A line intersecting a circle in two points is called a _______. *

(a) Secant (b) Chord (c) Diameter (d) Tangent

6. The distance between two parallel tangents to a circle of radius 5cm is: **

(a). 10cm (b) 11cm (c) 12cm (d) 14cm

7. If the circumference of a circle increases from 4π to 8π, then its area will become **

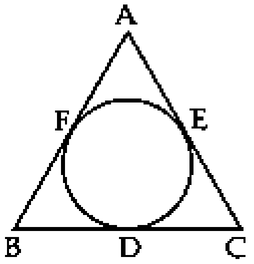
(a) half (b) 2 times (c) 4 times (d) does not change

8. Number of tangents drawn at a point of the, circle is/are***

(a) one (b) two (c) none (d) infinite

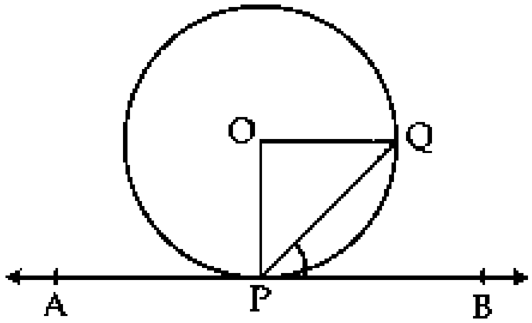
9. In the fig. if the semi perimeter of ΔABC = 23cm, then AF + BD + CE is: ***

a) 46cm b)11.5cm c)23cm d)34.5cm



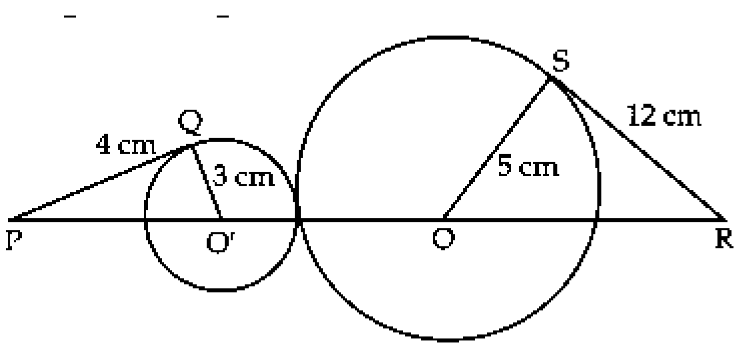
10. APB is a tangent to a circle with centre O, at point P. If $< QPB = 50^0$, then the measure of $<POQ$ is: **

a) 120^0 b) 100^0 c) 140^0 d) 95°



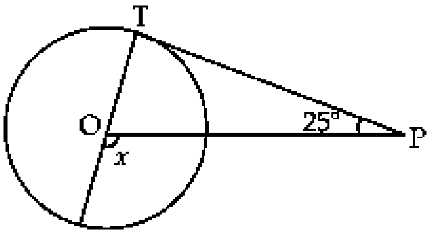
11. In fig. the length of PR is: ***

a) 20cm b)26cm c) 24cm d) 28cm



12. In fig. PT is a tangent to a circle with centre O and $< TPO = 25^0$, then the measure of x is: **

a) 120^0 125^0 c) 110^0 d) 115^0

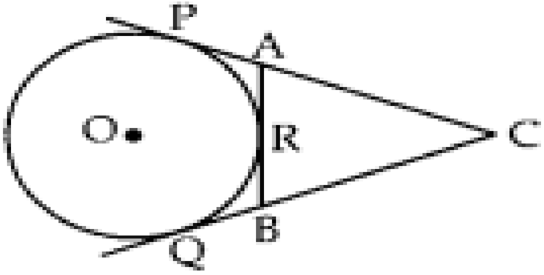


13. Maximum number of common tangents that can be drawn to two circles intersecting at two distinct points is: *

a) 1 b) 2 c) 3 d) 4

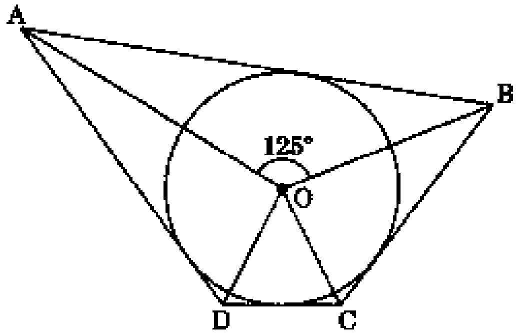
14. In given figure, CP and CQ are tangents to a circle with centre O. ARB is another tangent touching the circle at R. If CP = 11 cm and BC = 6 cm then the length of BR is***

(a) 6 cm (b) 5 cm
(c) 4 cm (d) 3 cm



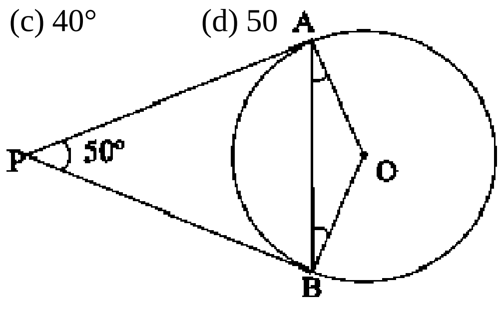
15. In Fig., if ∠AOB = 125°, then ∠COD is equal to?**

(a) 62.5° (b) 35° (b) 45° (d) 55°



16. In Fig., if PA and PB are tangents to the circle with centre O such that ∠APB = 50°,then ∠OAB is equal to? **

(a) 25° (b) 30° (c) 40° (d) 50



ASSERTION AND REASONING BASED QUESTIONS

Q.17. Assertion (A): If in a circle, the radius of the circle is 3 cm and distance of a point from the centre of a circle is 5 cm, then length of the tangent will be 4 cm.

Reason (R): (hypotenuse) 2 = (base) 2 + (height) 2

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q.18. Assertion (A): The two tangents are drawn to a circle from an external point, then they subtend equal angles at the centre. ***

Reason (R):A parallelogram circumscribing a circle is a rhombus.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q-19. Assertion (A): PA and PB are two tangents to a circle with centre O. Such that $\angle AOB = 110°$, then $\angle APB = 90°$. **

Reason (R): The length of two tangents drawn from an external point are equal.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q-20 Assertion (A): The length of the tangent drawn from a point 8cm away from the centre of circle of radius 6cm is 2√7 cm. **

Reason (R): If the angle between two radii of a circle is 130° , then the angle between the tangents at the end points of radii at their point of intersection is 50° .

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q.21. Assertion (A): If a chord AB subtends an angle of 60°. at the centre of a circle, then the angle between the tangents at A and B is also 60° .

Reason (R): The length of the tangent from an external points P on a circle with centre O is always less than OP.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

ONE MARK QUESTIONS (OTHER THAN MCQs)

22. Tangent to a circle intersects the circle at----------- point(s)? **

23.The tangent at any point of circle is perpendicular to the----------through the point of contact.

24. The lengths of tangents drawn from an external point to a circle are not equal. (true/false) **

25. If diagonal of a cyclic quadrilateral are the diameters of a circle through the vertices of a quadrilateral, then quadrilateral is a ----------.

26. Given three non collinear points, then the number of circles which can be drawn through these three points are? **

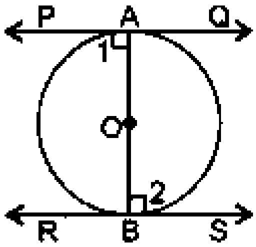
27. PQ is a tangent drawn from an external point P to a circle with centre O and QOR is thediameter of the circle. If $\angle POR = 120^0$, what is the measure of $\angle OPQ$?

SECTION B

SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

EXAMPLE-1. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Solution:



Proof: ∠1 = 90° …(i)

∠2 = 90° …(ii)

∠1 = ∠2 … [From (i) & (ii)

But these are alternate interior angles

∴PQ || RS   Hence proved.

EXAMPLE-2. From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB.

Answer:

∵ PA and PB are tangents to the given circle.

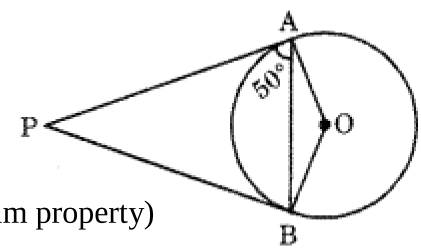
∴ ∠PAO = 90° (Radius is perpendicular to the tangent at the point of contact O.)

Now, ∠PAB = 50° (Given)

∴ ∠OAB = ∠PAO – ∠PAB

= 90° – 50° = 40°

In Δ OAB OB = OA (Radii of the circle)



∴ ∠OAB = ∠OBA = 40°

(Angles opposite to equal sides are equal)

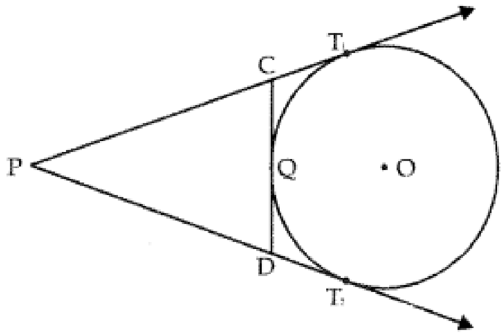
Now, ∠AOB + ∠OAB + ∠OBA = 180° (Angle sum property)

⇒ ∠AOB = 180° – (40° + 40°) = 100°

EXAMLPE-3. In fig., PT_1 and PT_2 are tangents to the circle drawn from an external point P. CD is a third tangent touching circle at Q. If PT_2 = 12 cm and CQ = 2 cm. What is the length of PC?

Answer:-

Length of tangents drawn from external point are equal.



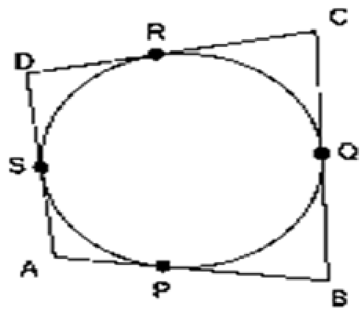
Therefore, $PT_1 = PT2 = 12$ cm

$CQ = CT_1 = 2$ cm

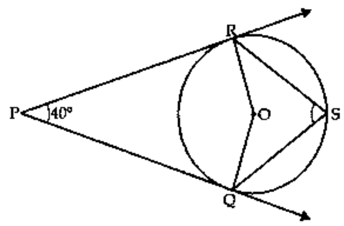
Now, $PC = PT_1 - CT_1 = (12 - 2)$ cm = 10 cm

ADDITIONAL SHORT ANSWER TYPE QUESTIONS -1 (2 MARKS EACH)

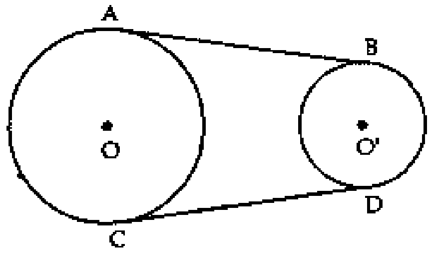
1. A quadrilateral ABCD is drawn to circumscribe a circle (see Fig.).
   Prove that AB + CD = AD + BC**



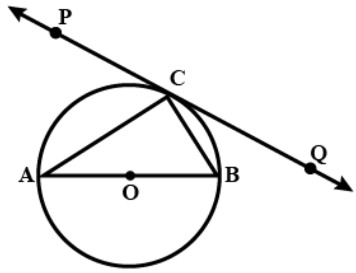
2. Prove that the lengths of tangents drawn from an external point to a circle are equal. ***
3. In fig., PQ and PR are tangents drawn from P. If ∠QPR = 40°, then find ∠QSR. *



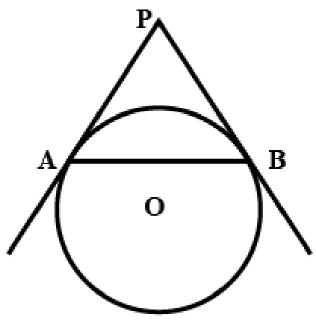
4. In the figure, AB and CD are common tangents to two circles of unequal radii.
   Prove that AB = CD. ***



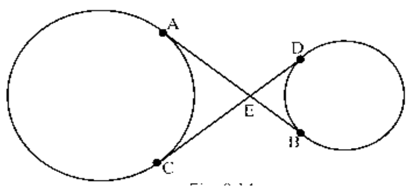
5. Prove that the line segments joining the points of contact of two parallel tangents is a diameter of the circle. *
6. PQ is a tangent at a point C to a circle with centre O. if AB is a diameter and ∠CAB=30°, find ∠PCA. *



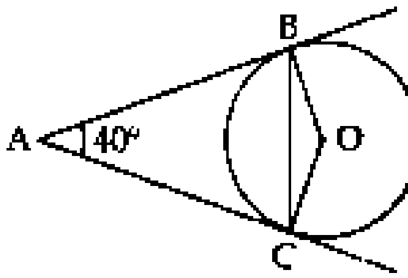
7. In figure, AP and BP are tangent with centre O, such that AP=5cm and ∠APB = 60°.Find the length of chord AB .



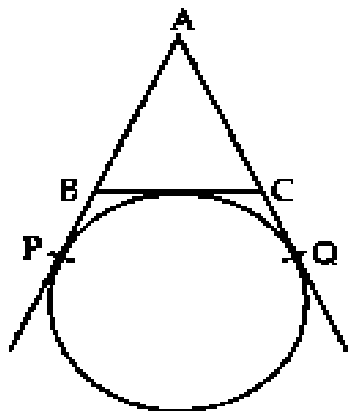
8. In Fig., common tangents AB and CD to two circles intersect at E. Prove that AB = CD*.



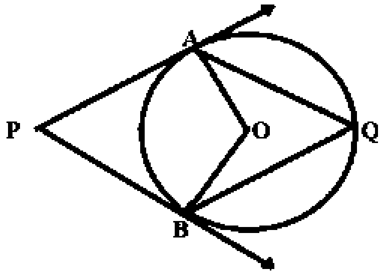
9. In the given figure, AB and AC are tangents to the circle with centre o such that ∠BAC = 40°. Then calculate ∠BOC. **



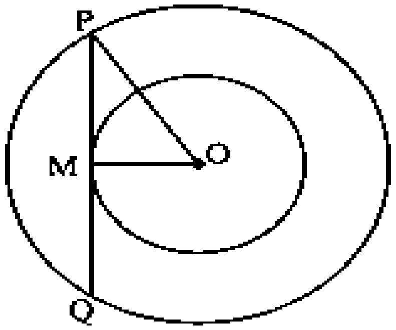
10. In the given figure, AP, AQ and BC are tangents to the circle. If AB = 5 cm, AC = 6 cm and BC = 4 cm, then calculate the length of AP (in cm).



11. In the given figure, O is the centre of circle. Find ∠AQB, given that PA and PB are tangents to the circleand ∠APB= 75°**.



12. Find the perimeter (in cm) of a square circumscribing a circle of radius a cm.
13. Two concentric circles are of radii 7 cm and r cm respectively, where r > 7. A chord of the larger circle, of length 48 cm, touches the smaller circle. Find the value of r. **



14. Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre. **
15. The length of a tangent from point A at a distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle. ***
16. A chord of a circle of radius 10 cm subtends a right angle at its centre. Calculate the length of the chord (in cm) **

Section C

SHORT ANSWER TYPE-2 (3 MARKS)

EXAMPLE 1. Two tangents segments PA and PB are drawn to a circle with centre O such that ∠APB = 120°. Prove that OP = 2 AP.

Solution:

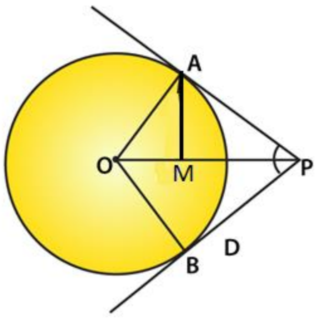
Given: From a point P. Outside the circle with centre O, PA and PB are tangents drawn and ∠APB = 120°

And, OP is joined.

To prove: OP = 2 AP

Construction: Take mid-point M of OP and join AM, and also join OA and OB.

Proof:



In the right ΔOAP,

∠OPA = 1/2∠APB = 1/2 (120°) = 60°

∠AOP = 90° – 60° = 30° [Angle sum property]

M is the mid-point of hypotenuse OP of ΔOAP [from construction]

So, MO = MA = MP

∠OAM = ∠AOM = 30° and ∠PAM = 90° – 30° = 60°

Thus, ΔAMP is an equilateral triangle

MA = MP = AP

But, M is the mid-point of OP

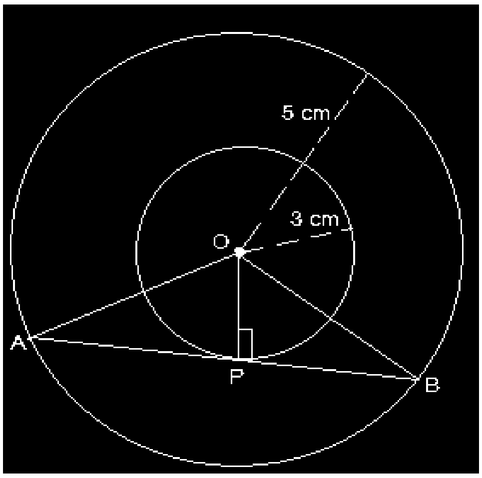
So,

OP = 2 MP = 2 AP

– Hence proved.

EXAMPLE -2. **Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.**

Answer:



Draw two concentric circles with the centre O. Now, draw a chord AB in the larger circle, which touches the smaller circle at a point P, as shown in the figure below.

From the above diagram, AB is tangent to the smaller circle to point P.

$\therefore OP \perp AB$

Using Pythagoras' theorem in triangle OPA,

$OA^2 = AP^2 + OP^2$

$5^2 = AP^2 + 3^2$

$AP^2 = 25 - 9$

$AP = 4$

Now, as $OP \perp AB$,

Since the perpendicular from the centre of the circle bisects the chord, AP will be equal to PB.

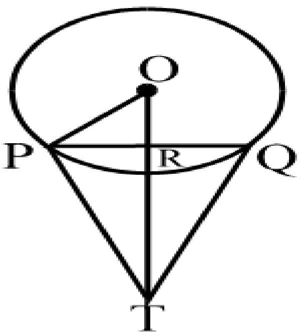
So, $AB = 2AP = 2 \times 4 = 8$ cm

So, the length of the chord of the larger circle is 8 cm.

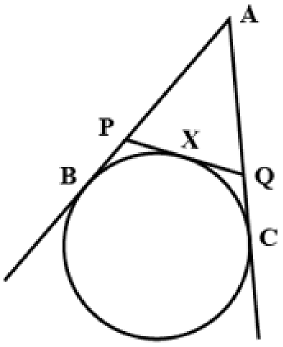
ADDITIONAL SHORT ANSWER TYPE-2 (3MARKS)

Q.1 Prove that a parallelogram circumscribing a circle is a rhombus. ***

Q.2 In the figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length TP. ***



Q.3 If AB, AC, PQ are tangents in the figure and AB = 5 cm. Find the perimeter of △ APQ. **



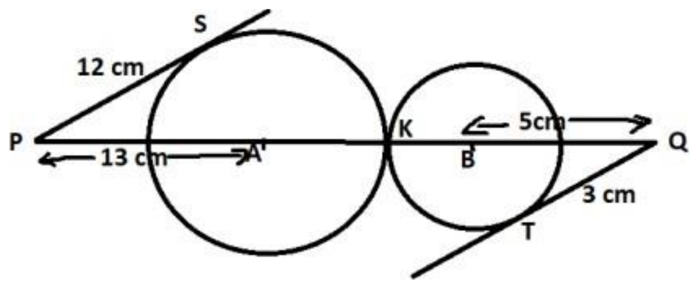
Q.4 Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠ PTQ = 2 ∠ OPQ. *

Q.5. Prove that tangent drawn at any point of a circle is perpendicular to the radius through the point of contact. ***

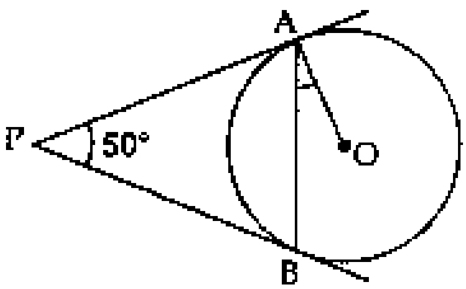
Q.6. From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle. Prove that OT is the right bisector of line segment PQ.

Q.7. Two tangents PQ and PR are drawn from an external point to a circle with centre O. Prove that QORP is a cyclic quadrilateral. *

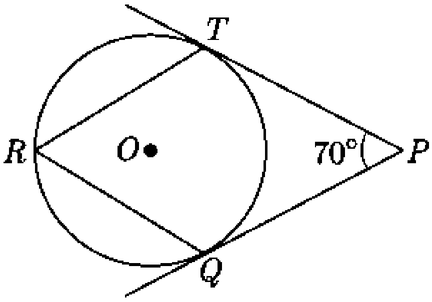
Q.8. In fig, two circles with centres A and B touch each other externally at K. find the length ofsegment PQ. (Given PA=13 cm , BQ=5 cm , PS=12 cm AND QT=3 cm) ***



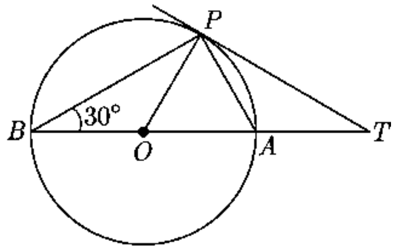
Q.9. In the given figure, PA and PB are tangents to the circle with centre O such that ∠APB = 50°. Write the measure of ∠OAB. **



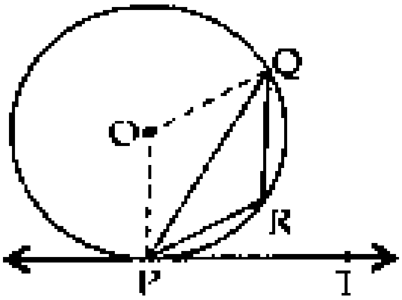
Q.10. In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, find ∠TRQ. *



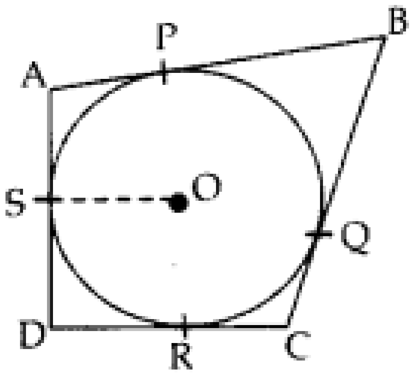
Q.11. In the given figure, BOA is a diameter of a circle and the tangent at a point P meets BA whenproduced at T. If ∠PBO = 30°, what is the measure of ∠PTA?



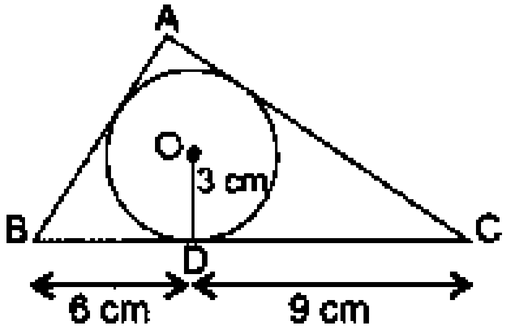
Q. 12. In the given figure, PQ is a chord of a circle with centre O and PT is a tangent. If ∠QPT = 60°, find ∠PRQ. **



Q. 13. In the given figure, a circle is inscribed in a quadrilateral ABCD touching its sides AB, BC, CD and AD at P, Q, R and S respectively. If the radius DA of the circle is 10 cm, BC = 38 cm, PB = 27 cm and AD ⊥ CD, then calculate the length of CD. ***



Q.14 In the figure, a ΔABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of ΔABC is 54 cm^2, then find the lengths of sides AB and AC. ***



Q. 15 If a, b, c are the sides of a right triangle where c is the hypotenuse, prove that the radius r of the circle which touches the sides of the triangle is given by r = (a + b - c)/2. ***

Q. 16. Prove that the tangents drawn at the end points of a chord of a circle make equal angles with the chord. *

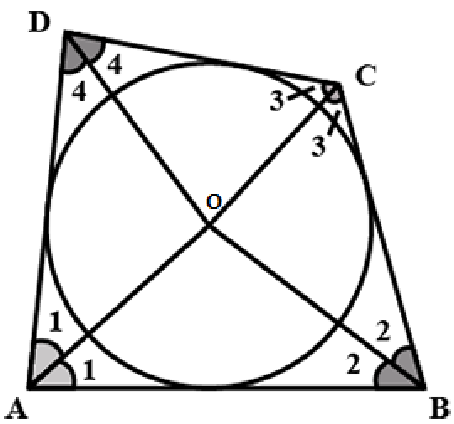
Q. 17 If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that AQ = 1/2 (BC + CA + AB) **

Section D

Long Answer (LA) type questions (5 marks each.)

EXAMPLE 1.

1. **Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.**



Let ABCD be a quadrilateral circumscribing a circle with centre O.

Now join AO, BO, CO, DO.

From the figure, $\angle DAO=\angle BAO$ [Since, AB and AD are tangents]

Let $\angle DAO=\angle BAO=1$

Also $\angle ABO=\angle CBO$ [Since, BA and BC are tangents]

Let $\angle ABO=\angle CBO=2$

Similarly we take the same way for vertices C and D

Sum of the angles at the centre is 360°

Recall that sum of the angles in quadrilateral, ABCD = 360°

$=2(\angle 1+\angle 2+\angle 3+\angle 4)=360°$

Or, $\angle 1+\angle 2+\angle 3+\angle 4=180°$

In ΔAOB, $\angle BOA=180°-(\angle 1+\angle 2)$

In ΔCOD, $\angle COD=180°-(\angle 3+\angle 4)$

$\angle BOA+\angle COD=360°-(\angle 1+\angle 2+\angle 3+\angle 4)$

$=360°-180°$

$=180°$

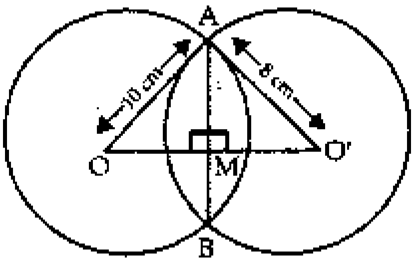
Since AB and CD subtend supplementary angles at O.

Thus, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

EXAMPLE 2. Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Find the distance between their centres.

Answer:-

Two circles having centre O and O' and OA = 10 cm; O'A = 8 cm respectively.



Also, AB = 12 cm be the length of common chord.

∴ AM = 12 = 12 (12) = 6 cm

In right angled ΔOMA,

$OA^2 = OM^2 + AM^2$

$(10)^2 = OM^2 + (6)^2$

or $OM^2 = 100 - 36$

or $OM^2 = 64 = (8)^2$

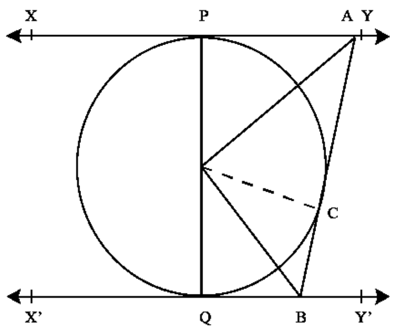
or OM = 8 cm

Now, in right Δ O'MA,

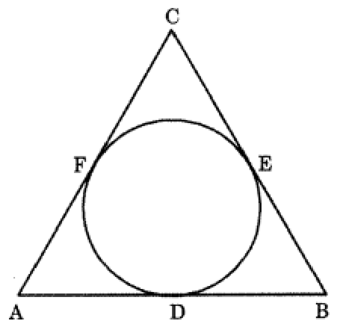
$O'A^2 = O'M^2 + AM^2$
$(8)^2 = O'M^2 + (6)^2$
or $O'M^2 = 64 – 36 = 28$
or $O'M = \sqrt{28} = 5.29$ cm.
∴ Required, distance between the centres
$= OO' = OM + MO'$
$= (8 + 5.29)$ cm
$= 13.29$ cm.

More Long Answer (LA) type questions

1. In the figure XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C interesting XY at A and X'Y' at B, what is the measure of ∠AOB. ***



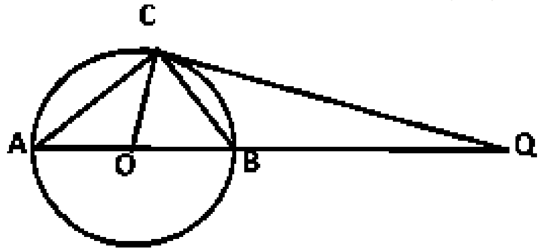
2. In figure, a circle is inscribed in a Δ ABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF. ***



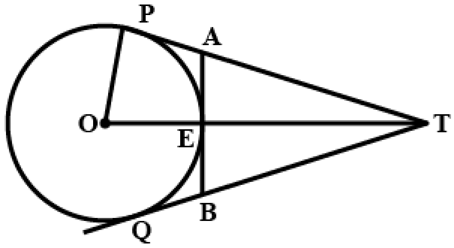
3. ABC is a right triangle, right angled at B. A circle is inscribed in it. The lengths of the two sides containing the right angle are 6 cm and 8 cm. Find the radius of the incircle. *

4. In fig AB is diameter of a circle with centre O and QC is a tangent to the circle at C. If

∠CAB=30°, find ∠CQA and ∠CBA.



5. In fig, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13cm and OT intersect circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle. **



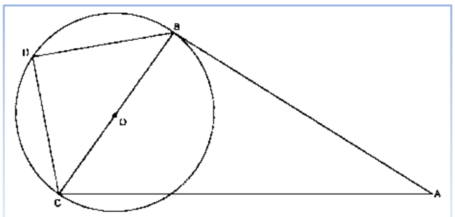
6. The figure below represents a circle with centre O and diameter 12cm

In triangle DBA, ∠DBC = ∠BCD and ∠A = 50°.

a. What is the measure of ∠DCA?

b. Sahil said that, "The quadrilateral DBAC is a cyclic quadrilateral." Is Sahil correct?Give a reason to support your answer.

c. In triangle BAC, the length of side CA = 2.5 times OB. What is the length of side BA?

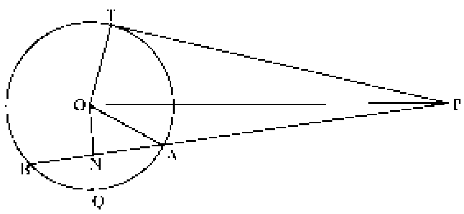


7. In following Fig. from an external point P, a tangent PT and a line segment PAB is drawn to a circle with centre O. ON is perpendicular on the chord AB. Prove that***

(i) PA . PB = $PN^2 - AN^2$

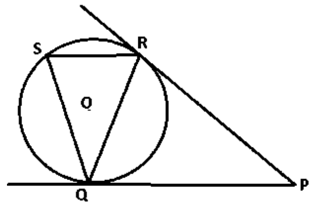
(ii) $PN^2 - AN^2 = OP^2 - OT^2$

(iii) PA.PB = PT^2



8. Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc. **

9. In fig, tangents PQ and PR are drawn from an external point P to a circle with centre O, such



that ∠RPQ= 30°. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS

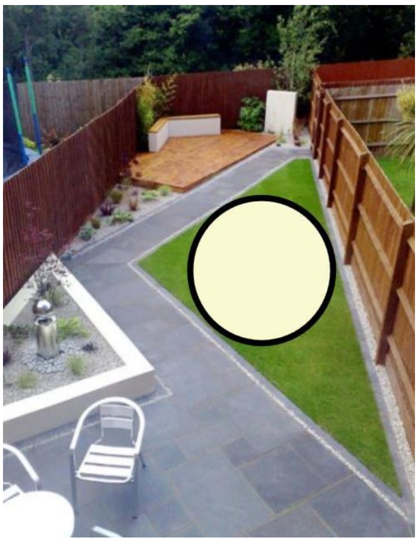
10. Two circles with centres O and O' of radii 3 cm and 4 cm, respectively intersect at two points P and Q such that OP and O'P are tangents to the two circles. Find the length of the common chord PQ. **

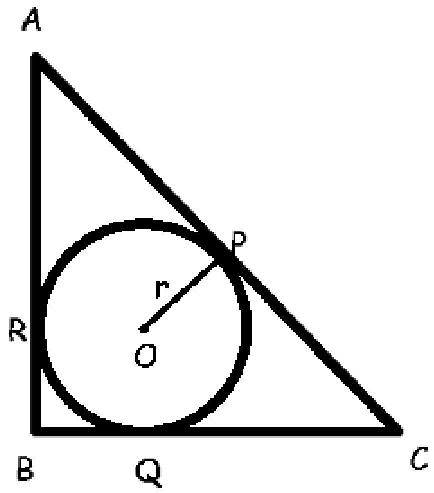
11. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC intowhich BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively . Find thesides AB and AC. **

CASE STUDY BASED QUESTIONS

CASE STUDY-1(PLAYGROUND)**

A playground is in the shape of a triangle with right angle at **B, AB** = **3** m and **BC** = **4** m. A pit was dig inside it such that it touches the walls **AC, BC** and **AB** at **P, Q** and **R**, respectively such that **AP** = x m.





Based on the above information, answer the following questions.

(i) The value of AR =

(a) 2x m (b) x / 2 m (c) x m (d) 3x m

(ii) The value of BQ=

(a) 2x m (b) (3 − x) m (c) (2 − x) m (d) 4x m

(iii) The value of CQ=

(a) (4 + x) m (b) (5 − x) m (c) (1 + x) m (d) Both (b) and (c)

(iv) Which of the following is correct?

(a) Quadrilateral AROP is a square (b) Quadrilateral BROQ is a square

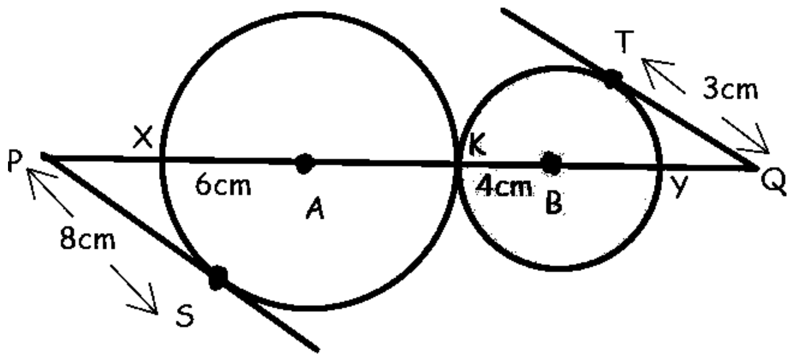
(c) Quadrilateral CQOP is a square (d) None of the above

(v) Radius of the pit is

(a) 1 m (b) 3 m (c) 4 m (d) 5 m

CASE STUDY – 2 (CIRCLE DRAWING)**

A student draws two circles that touch each other externally at point **K** with centres **A** and **B** and radii 6 cm and 4cm respectively as shown in the figure



Based on the above information, answer the following questions.

(i) The value of PA =

(a) 10 cm (b) 5 cm (c) 13 cm (d) Can't be determined

(ii) The value of BQ=

(a) 4 cm (b) 5 cm (c) 6 cm (d) 18 cm

(iii) The value of PK =

(a) 13 cm (b) 15 cm (c) 16 cm (d) 18 cm

(iv) The value of QY =

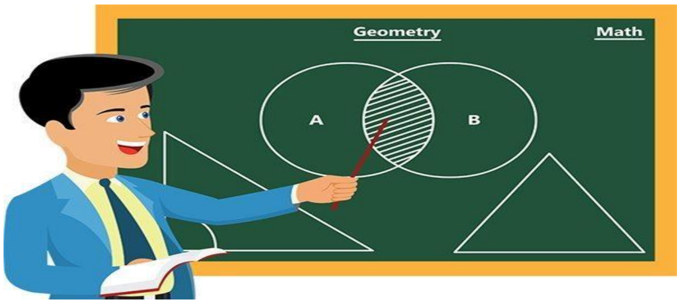
(a) 2 cm (b) 5 cm (c) 1 cm (d) 3 cm

(v) If two circles touch externally, then the number of common tangents can be drawn is

a) 1 (b) 2 (c) 3 (d) None of these

CASE STUDY – 3**

Kuldeep loves geometry. So, he was curious to know more about the concepts of circles. His grandfather is a mathematician. So, he reached to his grandfather to learn something interesting about tangents and circles. His grandfather gave him knowledge on circles and tangents and ask him to solve the following questions



(i) In the given figure, AP, AQ and BC are tangents to the circle such that AB = 7 cm, BC = 4cm and AC = 9 cm. Find AP

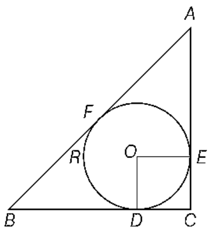
(a) 12 cm (b) 15 cm

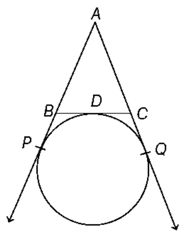
(c) 13 cm (d) 10 cm

(ii) A circle of radius 3 cm is inscribed in a right angled ΔBAC such that BD = 9 cmand DC= 3 cm Find the length of AB.

(a) 6 cm (b) 12 cm

(c) 15 cm (d) 10 cm





(iii) In the given figure, what is the length of CD?

(a) 11 cm (b) 9 cm

(c) 7 cm (d) 13 cm

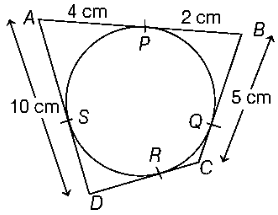
(iv) If PA and PB are two tangents to a circle with centre O from an external point P such that ∠OPB = 50°, then find ∠BPA

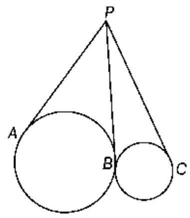
(a) 60° (b) 50° (c) 120° (d) 100°

(v) In the given figure, P is an external point from, which tangents are drawnto two externally touching circles. If PA =11 cm, then find PC.

(a) 3.5 cm (b) 4 cm

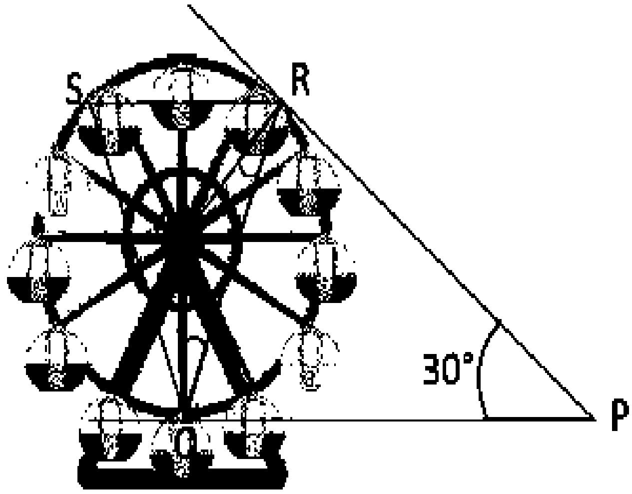
(c) 11 cm (d) Can’t be determined

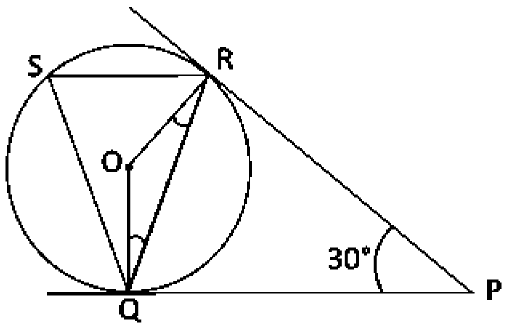




CASE STUDY-4 ***

A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passengers carrying components (commonly referred to as passenger cars,cabins, tubs, capsules, gondolas, or pods) attached to the rim in such a way that as the wheel turns, they are kept upright, usually by gravity. After taking a ride in Ferris wheel, Aarti came out from the crowd and was observing her friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given below.





(i) In the given figure find ∠ROQ.

(A) 60° (B) 100° (C) 150° (D) 90°

(ii) Find ∠RQP.

(A) 75° (B) 60° (C) 30° (D) 90°

(iii) Find ∠RSQ.

(A) 60° (B) 75° (C) 100° (D) 30°

(iv) Find ∠ORP.

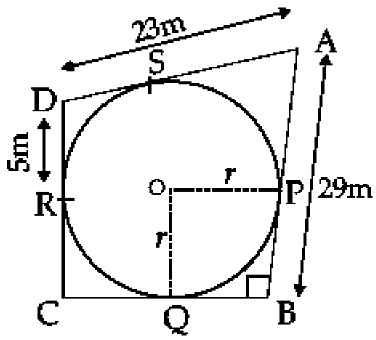
(A) 90° (B) 70° (C) 100° (D) 60°

(v) If PQ=40m and OQ=30m then PO=

(A) 50m (B) 60m (C) 70m (D) 80m

CASE STUDY-5*

ABCD is a playground. Inside the playground a circular track is present such that it



touches AB atpoint P, BC at Q, CD at R and DA at S.

1. If DR = 5 m, then DS is equal to:

(A) 6 m (B) 11 m (C) 5 m (D) 18 m

2. The length of AS is:

(A) 18 m (B) 13 (C) 14 m (D) 12 m

3. The length of PB is:

(A) 12 m (B) 11 m (C) 13 m (D) 20 m

4. What is the angle of OQB?

(A) 60° (B) 30° (C) 45° (D) 90°

5. What is the diameter of given circle?

(A) 22 m (B) 33 m (C) 20 m (D) 30 m

CASE STUDY- 6 **

Varun has been selected by his School to design logo for Sports Day T-shirtsfor students and staff. The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches thesides AB, BC and CA at points D, E and F respectively. The lengths of sidesAB, BC and CA are 12 cm, 8 cm and 10 cm respectively.

i) Find the length of AD

a) 7 b) 8 c) 5 d) 9

ii) Find the Length of BE

a) 8 b) 5 c) 2 d) 9

iii) Find the length of CF

a) 9 b) 5 c) 2 d) 3

iv) If radius of the circle is 4cm, Find the area of Δ OAB:

a) 20 b) 36 c) 24 d) 48

v) Find area of ΔABC

a) 50 b) 60 c) 100 d) 90

CASE STUDY-7*

People of village want to construct a road nearest to the circular village Parli. The road cannot pass through the village. But the people want the road should be at the shortest distance from the center of the village. Suppose the road start from point O which is outside the circular village andtouch the boundary of the circular village at point A such that OA = 20 m. And also, the straight distance of the point O from the center C of the village is 25 m.

i) Find the shortest distance of the road from the centre of the village a) 15m b) 14m c) 13m d) 12m

ii) Which method should be applied to find the shortest distance?
a)Concept of tangent to a circle b) Pythagoras theorem c)Both a and b
d) None of these

iii) If a point is inside the circle, how many tangents can be drawn from that point
a) 0 b) 1 c) 2 d) 3

iv) Number of common tangents can be drawn to two circles which do not intersect
a) 2 b) 3 c) 4 d) 1

v) If we draw two tangents at the end of the diameter, these tangents are always
a)Parallel b) perpendicular c) coincident d) None of these

CASE STUDY-8***

Three girls Reshma, Salma, and Mandeep are playing a game by standing on a circle. Reshma throws a ball to Salma, Salma to Mandeep, Mandeep to Reshma. If the distance between Reshma and Mandeep is 6 m, and between Reshma and Salma is 8 m. Mandeep and Salma are standing at ends of diameter and if O is the centre of the circle, then,

i) Length of the longest chord of the circle
a) 6m b) 8 m c)10m d) 12m

ii) Find the diameter of the circle.
a) 6m b) 8 m c)10m d) 12m

iii) The radius of the circle is.
a) 5m b) 8 m c)11m d) 15m

iv) Measure of <MRS is.
a) 180° b) 90° c)120° d) 100°

v) What is the distance between Reshma and Mandeep?
a) 9m b) 4.8 m c) 9.6m d) 10.6 m

ANSWER KEY: MCQ BASED QUESTIONS (1 MARK)

Q. NO.	ANSWER	Q. NO.	ANSWER
1	B	16	A
2	C	17	A
3	B	18	B
4	B	19	D
5	A	20	B
6	A	21	D
7	C	22	ONE
8	A	23	RADIUS
9	C	24	FALSE
10	B	25	RECTANGLE
11	B	26	ONE
12	D	27	30°
13	B		
14	B		
15	D		

ANSWER KEY: ADDITIONAL SHORT ANSWER TYPE QUESTIONS -1 (2 MARKS)

Q. NO.	ANSWER	Q. NO.	ANSWER
1	SELF	9	140°
2	SELF	10	7.5 CM
3	70 °	11	52.5°
4	SELF	12	8a cm
5	SELF	13	r=25 CM
6	60°	14	SELF
7	5 CM	15	RADIUS=3 CM
8	SELF	16	AB = $10\sqrt{2}$ cm

ANSWER KEY: ADDITIONAL SHORT ANSWER TYPE QUESTIONS -2 (3 MARKS)

Q. NO.	ANSWER	Q. NO.	ANSWER
1	SELF	9	25°
2	20/3	10	∠TRQ=55º
3	10CM	11	∠PTA=30°
4	SELF	12	∠PRQ. =120°
5	SELF	13	21CM
6	SELF	14	AB=9CM,AC=12CM
7	SELF	15	SELF
8	27CM	16	SELF
		17.	SELF

ANSWER KEY:MORE LONG ANSWER TYPE QUESTIONS (5 MARKS)

Q. NO.	ANSWER	Q. NO.	ANSWER
1	90°	9	30°
2	AD=7CM,BE=5CM,CF=3CM	10	4.8 cm
3	2cm	11	AB=15CM, AC=13CM
4	30° AND 60°	12	8√2cm
5	6.6CM	13	70°
6	a)85° , b)NO, c)1.5 TIMES,OB=18 CM	14	1/3
7	SELF	15	SELF
8	SELF	16	4√10CM
		17.	SELF

ANSWER KEY: CASE STUDY BASED QUESTIONS

CASE STUDY-1 (PLAYGROUND)					
QUESTIONS	I	II	III	IV	V

ANSWER	(C) x m	(b)(3 − x) m	(d) Both b and c	(b) Quadrilateral BROQ is a square	(a) 1 m
CASE STUDY-2 (CIRCLE DRAWING)	(a) 10 cm	(b) 5 cm	(c) 16 cm	(c) 1 cm	(c) 3
CASE STUDY-3	(d) 10 cm	(c) 15 cm	(b) 9 cm	(d) 100°	(c) 11 cm
CASE STUDY-4 (FERRIS WHEEL)	(C) 150°	(A) 75°	(B) 75°	(A) 90°	(A) 50m
CASE STUDY-5 (PLAYGROUND)	(C) 5 m	(A) 18 m	(B) 11 m	(D) 90°	(A) 22 m
CASE STUDY-6 (SPORTS DAY T-SHIRT)	a) 7	b) 5	d) 3	c) 24	b) 60
CASE STUDY-7	a) 15m	c) both a andb	a) 0	c) 4	a) Parallel
CASE STUDY-8	c) 10m	c) 10m	a) 5m	b) 90°	c) 9.6m

******

CHAPTER -12

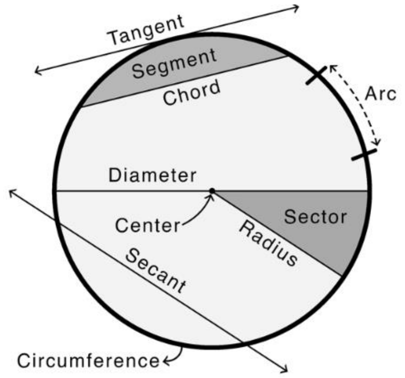
AREAS RELATED TO CIRCLE

Prepared by, BERNARD I. PURTY
TGT (MATHS)
KV LATEHAR

MAIN CONCEPTS

A closed, round geometric figure in which the set of all the points in the plane is equidistant from a given point called 'centre'.

Parts of a circle:-



Perimeter of a Circle:- The distance covered by travelling once around a circle is its perimeter, usually called its circumference. It is $\mathbf{2\pi r}$.

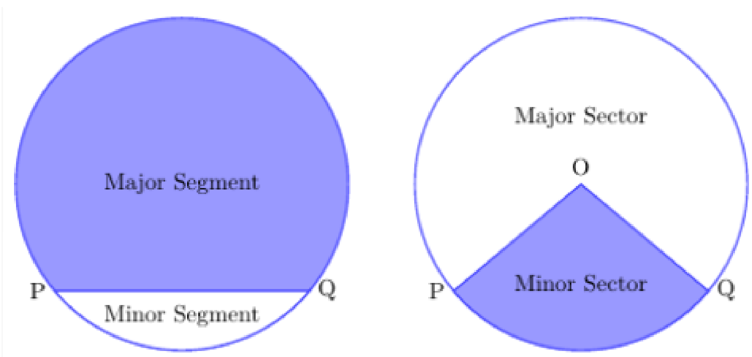
Area of a Circle:- The area enclosed by a circle of radius r is $\boldsymbol{\pi r^2}$.

- $\boldsymbol{\pi}$ **is equal to $\frac{22}{7}$ or 3.14 depending on the given question.**

Area of the circular path formed by two concentric circles of radii r_1 and r_2 ($r_1 > r_2$)

$$= \pi r_1^2 - \pi r_2^2 = \pi(r_1^2 - r_2^2)$$

Sector and Segment of a Circle:-

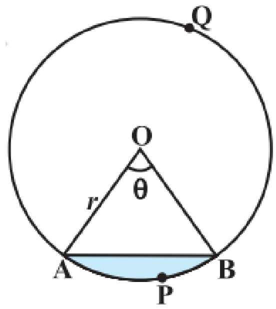


- **Area of the minor sector of angle θ =** $\frac{\theta}{360} \times \pi r^2$
- **Area of major sector of angle θ =** πr^2 – Area of the minor sector
- **Length of an arc of a sector of angle θ =** $\frac{\theta}{360} \times 2\pi r$

where r is the radius of the circle and θ the angle of the sector in degrees.

- **θ is called the central angle.**

Areas of Segment of a Circle:-



- **Area of the segment APB =** Area of the sector OAPB – Area of Δ OAB

$$= \frac{\theta}{360} \times \pi r^2 - \text{area of OAB}$$

- **Area of major segment AQB =** πr^2 – Area of the minor segment APB

SOLVED QUESTIONS

*****1. If the perimeter of the circle and square are equal, then the ratio of their areas will be equal to:**

(a) 14:11 (b) 22:7 (c) 7:22 (c) 11:14

Ans:- **(a) 14:11**

Explanation: Given,

The perimeter of circle = perimeter of the square

$2\pi r = 4a$

$a = \pi r/2$

Area of square = $a^2 = (\pi r/2)^2$

$A_{circle}/A_{square} = \pi r^2/(\pi r/2)^2$

= 14/11

***2. The area of a sector of a circle with radius 6 cm if the angle of the sector is 60° will be.**

(a) 142/7cm^2 **(b) 152/7 cm^2** **(c) 132/7 cm^2** **(d) 122/7 cm^2**

Ans:- **(c) 132/7 cm^2**

Explanation: Angle of the sector is 60°

Area of sector = (θ/360°) × π r^2

∴ Area of the sector with angle 60° = (60°/360°) × π r^2 cm^2

= (36/6) π cm^2

= 6 × (22/7) cm^2

= 132/7 cm^2

***3. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. The length of the arc is;**

(a) 20cm (b) 21cm (c) 22cm (d) 25cm

Ans”- **(c) 22cm**

Explanation: Length of an arc = (θ/360°) × (2πr)

∴ Length of an arc AB = (60°/360°) × 2 × 22/7 × 21

= (1/6) × 2 × (22/7) × 21

Or, Arc AB Length = 22cm

***4. If the radius of a circle is 4.2 cm, compute its area and circumference.**

Ans:- Area of a circle = πr^2

So, area = π(4.2)^2 = 55.44 cm^2

Circumference of a circle = 2πr

So, circumference **= 2π(4.2) = 26.4 cm**

****5. Find the area of the sector of a circle with a radius of 4cm and of angle 30°. Also, find the area of the corresponding major sector.**

Ans :- Radius = r = 4 cm, θ=30°
Area of sector = [θ/360]×πr^2
= 30/360×3.14×(4)^2
= 1/12×3.14×4×4
= 1/3×3.14×4
= 12.56/3 cm^2
= 4.19 cm^2
Area of major sector = ((360 − θ)/360)×πr^2

$= ((360 - 30))/360 \times 3.14 \times (4)^2$
$= 330/360 \times 3.14 \times 4 \times 4$
$= 11/12 \times 3.14 \times 4 \times 4$
$= 46.05\ cm^2$

*****6. A wheel has diameter 84 cm. Find how many complete revolutions must it take to cover 792 meters.**

Ans:- Let r be the radius of the wheel. Then,

Diameter = 84 cm

2r=84

r=42 cm

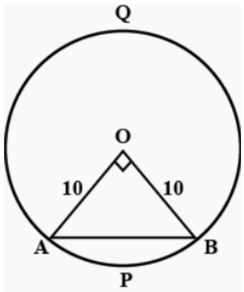
Circumference of the wheel = $2\pi r$=264cm=2.64m

So, the wheel covers 2.64 meters in one complete revolution.

Total number of revolutions in covering 792 meters = 792/2.64=300.

Hence, the wheel takes 300 revolutions in covering 792 meters.

****7. A chord AB of a circle of radius 10 cm makes a right angle at the centre of the circle. Find the area of the minor and major segments.**



Ans:- Area of segment APBA = area of sector OAPB−area of ΔOAB

$=\theta/(360°) \times \pi r^2 - 1/2 \times OB \times OA$

$=(90°)/(360°) \times 22/7 \times 10 \times 10 - 1/2 \times 10 \times 10$

=550/7-50 = (550-530)/7 = 200/7

$=28.5 cm^2$

Area of major segment = $\pi r^2 - 28.5 = 22/7 \times 10 \times 10 - 28.5$

= 2200/7−28.5=(2200−199.5)/7=200.5/7

$= 285.5\ cm^2$

*****8. If a square is inscribed in a circle, find the ratio of the areas of the circle and the square.**

Ans:- Let ABCD be a square inscribed in a circle of radius 'r'. Now, the diameter of circle is the diagonal of square.

Therefore, BD=2r. In △BDC, using Pythagoras theorem

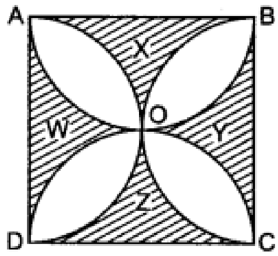
$BC^2+CD^2=BD^2 \Rightarrow a^2+a^2=(2r)^2 \Rightarrow 2a^2=4r^2 \Rightarrow a^2=2r^2$

∴Area of square=$2r^2$

Area of circle=πr^2

Required ration = $\pi r^2 : 2r^2 = \pi : : 2$

*****9. In figure, ABCD is a square of side 14 cm. Semi-circles are drawn with each side of square as diameter. Find the area of the shaded region.**



Ans:- Area of the square ABCD = 14 x 14 = 196 cm²

Area of semicircle AOB=1/2 x πr^2

=1/2×22/7x7x7

Similarly, area of semicircle DOC = 77 cm²

Hence, the area of shaded region (Part W and Part Y) = Area of square -Area of two semicircles AOB and COD

= 196 – 154 = 42 cm²

Therefore, area of four shaded parts (i.e. X, Y, W, Z) = (2 x 42) cm² = 84 cm²

****10. A chord subtends an angle of 90°at the centre of a circle whose radius is 20 cm. Compute the area of the corresponding major segment of the circle.**

Ans :- Point to note:

Area of the sector = $\theta/360 \times \pi \times r^2$

Base and height of the triangle formed will be = radius of the circle

Area of the minor segment = area of the sector – area of the triangle formed

Area of the major segment = area of the circle – area of the minor segment

Now,

Radius of circle = r = 20 cm and

Angle subtended = θ = 90°

Area of the sector = $\theta/360 \times \pi \times r^2 = 90/360 \times 22/7 \times 20^2$

Or, area of the sector = 314.2 cm^2

Area of the triangle = ½ × base × height = ½ × 20 × 20 = 200 cm^2

Area of the minor segment = 314.2 – 200 = 114.2 cm^2

Area of the circle = $\pi \times r^2 = (22/7) \times 20^2 = 1257.14$

Area of the major segment = 1257.14 – 114.2 = 1142 .94 cm^2

So, the area of the corresponding major segment of the circle = 1142 .94 cm^2

*****11. Calculate the perimeter of an equilateral triangle if it inscribes a circle whose area is 154 cm^2**

Ans: - Here, as the equilateral triangle is inscribed in a circle, the circle is an incircle.

Now, the radius of the incircle is given by,

r = Area of triangle/semi-perimeter

In the question, it is given that area of the incircle = 154 cm^2

So, $\pi \times r^2 = 154$

Or, r = 7 cm

Now, assume the length of each arm of the equilateral triangle to be "x" cm

So, the semi-perimeter of the equilateral triangle = (3x/2) cm

And, the area of the equilateral triangle = $(\sqrt{3}/4) \times x^2$

We know, r = Area of triangle/semi-perimeter

So, $r = [x^2(\sqrt{3}/4)/ (3x/2)]$

=> $7 = \sqrt{3}x/6$

Or, $x = 42/\sqrt{3}$

Multiply both numerator and denominator by $\sqrt{3}$

So, $x = 42\sqrt{3}/3 = 14\sqrt{3}$ cm

Now, the perimeter of an equilateral triangle will be = $3x = 3 \times 14\sqrt{3} = 72.7$ cm.

*****12. The wheels of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?**

Ans :- The radius of car's wheel = 80/2 = 40 cm (as D = 80 cm)

So, the circumference of wheels = $2\pi r = 80\pi$ cm

Now, in one revolution, the distance covered = circumference of the wheel = 80π cm

It is given that the distance covered by the car in 1 hr = 66km

Converting km into cm we get,

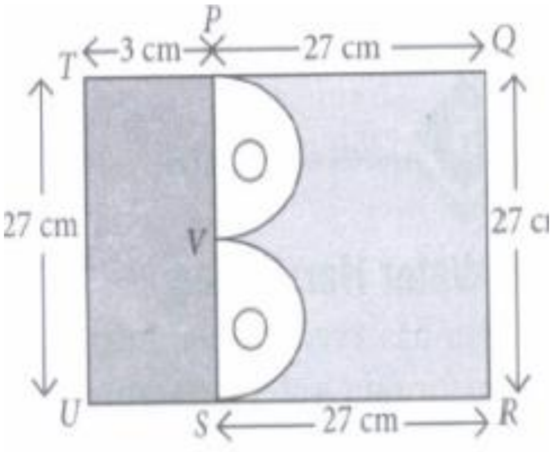
Distance covered by the car in 1hr = (66×10^5) cm

In 10 minutes, the distance covered will be = $(66 \times 10^5 \times 10)/60 = 1100000$ cm/s

$\therefore$ Distance covered by car = 11×10^5 cm

Now, the no. of revolutions of the wheels = (Distance covered by the car/Circumference of the wheels) = 11 × 105 /80 π = 4375.

*****13. Mr Ramanand purchased a plot QRUT to build his house. Following is scaled down figure of his plot. He leave space of two congruent semicircles for gardening and a rectangular area of breadth 3 cm for car parking.**



Based on the above information, answer the following questions. Use $\pi = 3.14$

(i) Area of square PQRS is

(a) 700 cm^2 (b) 729 cm^2 (c) 732 cm^2 (d) 735 cm^2

Ans:- (b) 729 cm^2. Area of square = 27 x 27 = 729cm^2

(ii) Area of rectangle left for car parking is

(a) 64 cm^2 (b) 76 cm^2 (c) 81 cm^2 (d) 100 cm^2

Ans:- (c) 81 cm^2. Area of Rectangle = 3x27= 81cm^2

(iii) Radius of semi-circle is

(a) 6.75 cm (b) 7 cm (c) 7.75 cm (d) 8.75 cm

Ans:- (a) 6.75 cm. Diameter of semi-circle = 27/2 = 13.5cm. Radius of circle = 13.5/2 = 6.75cm

(iv) Find the area of the shaded region

(a) 660.82 cm^2 (b) 666.72 cm^2 (c) 669.89 cm^2 (d) 700 cm^2

Ans:- (b) 666.72 cm^2. Area of shaded part= area of plot – area of two semi- circles.

= 27x30 – 3.14 x $(6.75)^2$ = 666.72cm^2

PRACTICE QUESTIONS

SECTION A

MULTIPLE CHOICE QUESTIONS:

**1. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is

(a) 2 units (b) π units (c) 4 units (d) 7 units

*2. Area of a sector of angle p (in degrees) of a circle with radius R is

(a) $p/180 \times 2\pi R$ (b) $p/180 \times \pi R^2$

(c) $p/360 \times 2\pi R$ (d) $p/720 \times 2\pi R^2$

***3. If the sum of the areas of two circles with radii R_1 and R_2 is equal to the area of a circle of radius R, then

(a) $R_1 + R_2 = R$ (b) $R_1^2 + R_2^2 = R^2$

(c) $R_1 + R_2 < R$ (d) $R_1^2 + R_2^2 < R^2$

***4. It is proposed to build a single circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park would be

(a) 10 m (b) 15 m (c) 20 m (d) 24 m

*5. The area of a quadrant of a circle with circumference of 22 cm is

(a) 77 cm^2 (b) 77/8 cm^2 (b) 35.5 cm^2 (c) 77/2 cm^2

**6. In a circle of radius 14 cm, an arc subtends an angle of 30° at the centre, the length of the arc is

(a) 44 cm (b) 28 cm (c) 11 cm (d) 22/3 cm

***7. The length of the minute hand of a clock is 14 cm. The area swept by the minute hand in 5 minutes is

(a) 153.9 cm^2 (b) 102.6 cm^2 (c) 51.3 cm^2 (d) 205.2 cm^2

***8. The radius of a circle whose circumference is equal to the sum of the circumferences of the two circles of diameters 36 cm and 20 cm is

(a) 56 cm (b) 42 cm (c) 28 cm (d) 16 cm

***9. The diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm respectively, is

(a) 31 cm (b) 25 cm (c) 62 cm (d) 50 cm

***10. The wheel of a motor cycle is of radius 35 cm. How many revolutions per minute must the wheel make so as to keep a speed of 66 km/h?

(a) 300 (b) 400 (c) 450 (d) 500

***11. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20m × 16m, then the area of the field in which the cow can graze is:

(a) 154 m^2 (b) 156 m^2 (c) 158 m^2 (d) 160 m^2

**12. A pendulum swings through on angle of 30∘ and describes an arc 8.8 cm in length. Find the length of pendulum in cm.

(a) 16.8 (b) 17.3 (c) 15.1 (d) 14.5

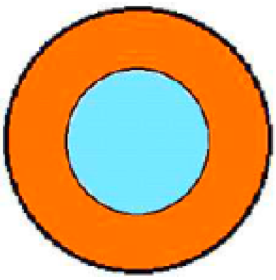
*13. Radius of the outer circle is 18 cm and the radius of the inner circle is 7 cm. What is the area of the region between the outer and the inner circles?

(a) 361 π cm^2 (b) 133 cm^2 (v) 192.5 cm^2 (d) 275 π cm^2

**14. A wire is bent to form a circle of radius 7 cm. From the resulting shape, a chunk of the wire is cut off, and the wire cut off measures 4 cm in length. The length of the remaining wire is

(a) 45cm (b) 50cm (c) 40cm (d) 42cm

*15. In the figure, the area of the outer ring iS



(a) Area of outer circle + Area of inner circle

(b) Area of outer circle – Area of inner circle

(c) Area of inner circle – Area of outer circle

(d) Area of outer circle

SECTION B

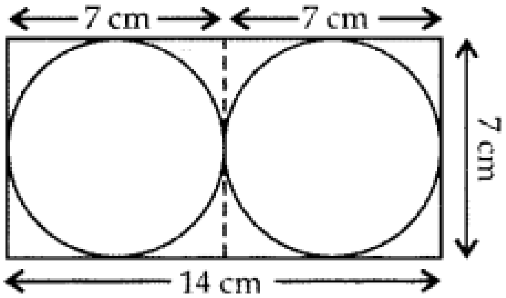
2 MARKS QUESTIONS:-

**1. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.

***2. If the area of a circle is equal to sum of the areas of two circles of diameters 10 cm and 24 cm, calculate the diameter of the larger circle (in cm).

***3. If the difference between the circumference and the radius of a circle is 37 cm, then using $\pi = 22/7$, calculate the circumference (in cm) of the circle.

**4. Observe the following figure

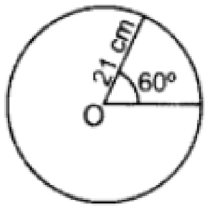


Two circular pieces of equal radii and maximum area, touching each other are cut out from a rectangular card board of dimensions 14 cm × 7 cm. Find the area of the remaining card board. [Use $\pi = 22/7$]

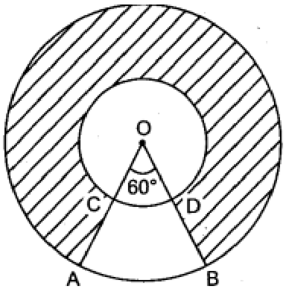
**5. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:

(i) the length of the arc

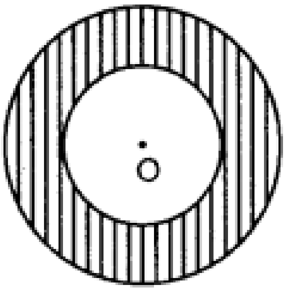
(ii) area of the sector formed by the arc. [Use $\pi = 22/7$]



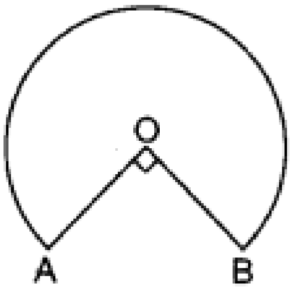
***6. In figure, two concentric circles with centre O, have radii 21 cm and 42 cm. If ∠AOB = 60°, find the area of the shaded region



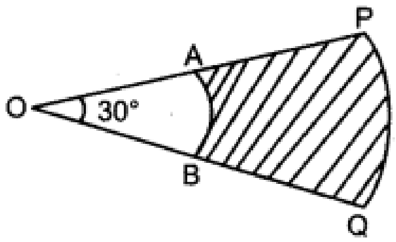
**7. In the given figure, the area of the shaded region between two concentric circles is 286 cm^2.If the difference of the radii of the two circles is 7 cm, find the sum of their radii.



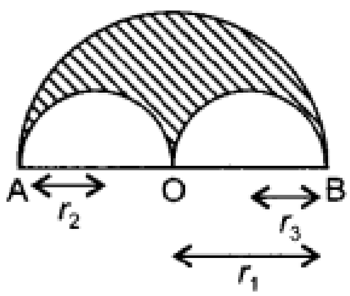
**8. In the given figure, the shape of the top of a table is that a sector of a circle with centre O and ∠AOB = 90°. If AO = OB = 42 cm, then find the perimeter of the top of the table



*9. In figure, PQ and AB are respectively the arcs of two concentric circles of radii 7 cm and 3.5 cm and centre O. If ∠POQ = 30°, then find the area of the shaded region.



***10. In given figure, a semicircle is drawn with O as centre and AB as diameter. Semicircles are drawn with AO and OB as diameters.If AB = 28 m, find the perimeter of the shaded region.



.

SECTION C

THREE MARKS QUESTIONS:

**1. A chord of length 10 cm divides a circle of radius $5\sqrt{2}$ cm in two segments. Find the area of the minor segment.

**2. Find the area of the major segment APB in a circle of radius 35 cm and ∠AOB = 90°.

***3. The circumference of a circle exceeds its diameter by 180 cm. Then find its radius.

***4. A boy is cycling such that the wheels of the cycle are making 140 revolutions per minute. If the diameter of the wheel is 60cm, calculate the speed in km/h.

***5. The circumference of a circular park is 314m. a 20m wide track runs around it. Calculate the cost of laying turf in the park at Rs. 1.25 per sq.m. and the cost of the concrete track at Rs1.25sp.m.

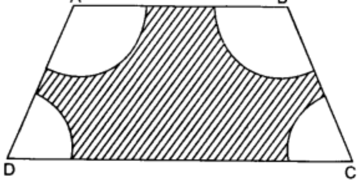
**6. A cow is tied with a rope of length 14 m at the corner of a rectangular field of dimensions 20m × 16m. Find the area of the field in which the cow cannot graze.

**7. A circular park is surrounded by a road 21m wide. If the radius of the park is 105m, find the area of the road.

**8. A circular pond is of diameter 17.5m is surrounded by a 2 m wide path. Find the cost of constructing the path at a rate of Rs 25 per m^2.

*9. Find the area of the sector of a radius 5cm, if the corresponding arc length is 3.5cm.

**10. In figure, ABCD is a trapezium with AB|| DC, AB=18cm, DC=32cm and the distance between AB and DC is t14cm. if arcs of eqal radii 7cm have been drawn, with centres A,B,C, and D, then find the shaded portion.

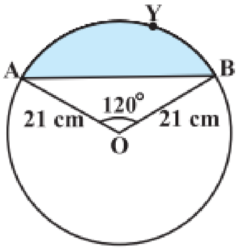


SECTION D

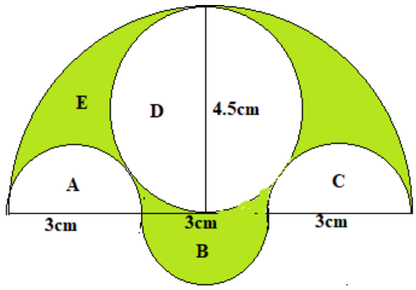
FIVE MARKS QUESTIONS:

***1. A round table cover has six equal designs like segments between two radii. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2. (Use$\sqrt{3} = 1.7$)

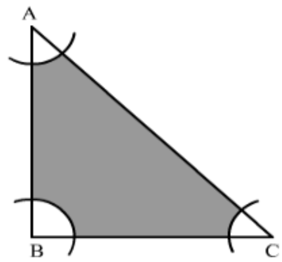
***2. Find the area of the segment AYB shown in Figure, if radius of the circle is 21 cm and ∠ AOB = 120°. (Use π = 22/ 7)



**3. In Fig. there are three semicircles, A, B and C having diameter 3 cm each, and another semicircle E having a circle D with diameter 4.5 cm are shown. Calculate the cost of painting the shaded region at the rate of ₹25 per cm^2

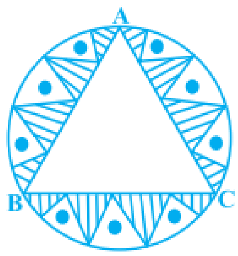


***4. With the vertices A, B and C of a triangle ABC as centres, arcs are drawn with radii 5 cm each as shown in Fig. 11.15. If AB = 14 cm, BC = 48 cm and CA = 50 cm, then find the area of the shaded region. (Use π = 3.14).



**5. A calf is tied with a rope of length 6 m at the corner of a square grassy lawn of side 20 m. If the length of the rope is increased by 5.5m, find the increase in area of the grassy lawn in which the calf can graze.

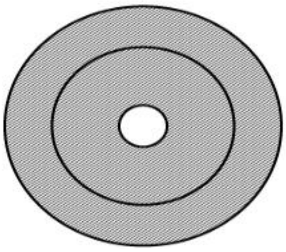
**6. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. Find the area of the design.



***7. Sides of a triangular field are 15 m, 16 m and 17 m. With the three corners of the field a cow, a buffalo and a horse are tied separately with ropes of length 7m each to graze in the field. Find the area of the field which cannot be grazed by the three animals.

***8. The diameters of front and rear wheels of a tractor are 80 cm and 2m respectively. Find the number of revolutions that rear wheel will make in covering a distance in which the front wheel makes 1400 revolutions.

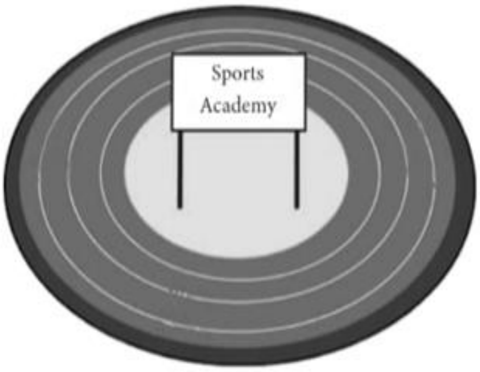
***9. An archery target has three regions formed by three concentric circles as shown in figure. If the diameters of the concentric circles are in the ratio 1:2:3, then find the ratio of the areas of three regions.



*10. Find the difference of the areas of the two segments of a circle formed by a chord of length 5cm subtending an angle of 90° at the centre.

CASE BASED QUESTIONS:

*****1. There is a race competition between all students of a sports academy, so that the sports committee can chose better students for a marathon. The race track in the academy is in the form of a ring whose inner most circumference is 264 m and the outer most circumference is 308m.**



Based on the above information, answer the following questions:-

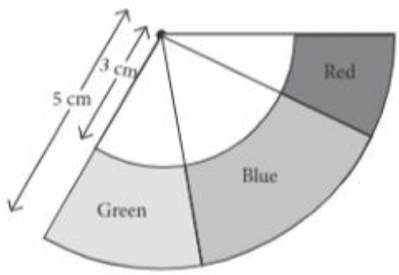
(i) Find the radius of the outermost circle.

(ii) Find the area of the race track.

(iii) If the cost of painting on the race track is Rs. 6 per m^2, then find the total cost for the painting of the whole race track.

****2. Sarita held a Japanese fan in her hand as shown in the figure below. It is shaped like a sector of a circle and made a thin material such as paper or feather. The inner and outer radii are 3cm and 5cm respectively. The fan has three colours i.e., red, blue, green.**

Based on the above information, answer the following questions:-

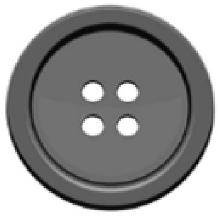


(i) If the region containing blue colour makes an angle of 80° at the centre, then find the area of the region having blue colour.

(ii) If the region containing green colour makes an angle of 60° at the centre, then find the area of the region having green colour.

(iii) Find the area of the whole region having radius 3cm if the central angle made by red region is 20°.

***3. While dusting a maid found a button whose upper face is of black colour as shown in the figure. The diameter of each of the smaller identical circles is ¼ of thee diameter of the larger circle whose radius is 16cm.**



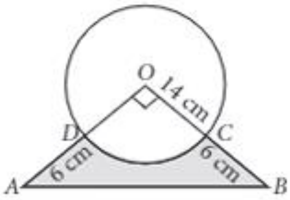
Based on the above information, answer the following questions:-

(i) Find the area of each of the smaller circle?

(ii) Find the area of black colour region?

(iii) Find the area of quadrant of the smaller circle?

*****4. Director of a company selects a round glass trophy for awarding their employees on annual function. Design of each trophy is made as shown in the figure, where it's base ABCD is golden plated from the front side at the rate of Rs 6 per cm^2.**



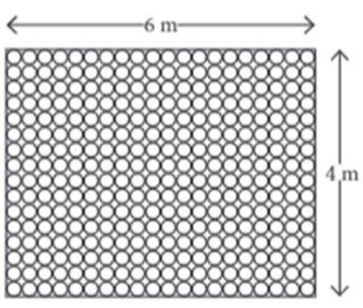
Based on the above information, answer the following questions:-

(i) Find the area of the sector ODCO.

(ii) Find the total cost of gold plating.

(iii) Find the length of arc DC.

****5. Shweta wants to change the design of the floor of her living room which is of the dimension 6m X 4m and it is covered with circular tiles of diameter 50cm each as shown below.**



Based on the above information, answer the following questions:-

(i) Find the number of circular tiles along length of room.

(ii) Find the total number of tiles in the floor.

(iii) Find the area of the floor that remains uncovered by the tiles.

ANSWWR KEY CHAPTER 12 AREA RELATED TO CIRCLE

SECTION A MCQ ANSWERS:-

1. (a) 2 units 2. (d) $p/720 \times 2\pi R^2$ 3. **(b)** $R_1^2 + R_2^2 = R^2$ 4. **(a) 10 m**

5. **(b) 77/8 cm^2** 6. **(d) 22/3 cm** 7. (c) 51.3 cm^2 8. (c) 28 cm

9 (d) 50 cm 10. (d) 500 11. (a) 154 m^2 12. (a) 16.8 13. (d) $275\ \pi$ cm^2

14. (c) 40cm 15. (b) Area of outer circle – Area of inner circle

SECTION B 2 MKS ANSWERS:-

1. 10cm 2. 26 cm 3. 44cm 4. 21cm^2 5. (i) 22cm (ii) 231cm^2

6. 3465cm^2 7. 13cm 8. 282cm 9. 9.625 cm^2 10. 88m

SECTION C 3 MKS ANSWERS:-

1. 14.25cm^2 2. 3500cm^2 3. 42cm 4. 15.84km/h 5. Rs.9812.5, Rs9420

6. 166 m^2 7. 15246 m^2 8. Rs 3061.50 9. 8.75cm^2 10. 196 m^2

SECTION D ANSWERS:-

1. Rs. 162.66 2. $\frac{21}{4}(88 - 21\sqrt{3})$ 3. Rs. 309 4. 296.75 cm^2 5. 75.625 m^2

6. $(\frac{22528}{7} - 768\sqrt{3})$ cm^2 7. $(24\sqrt{21}-77)$m^2 8. 560 9. 1:3:5

10. 32.1429 cm^2

CASE STUDY BASED ANSWERS:-

1. (i) 49m (ii) 2002m^2 (iii) Rs. 12012

2. (i) 11.17cm^2 (ii) 8.38cm^2 (iii) 12.57 cm^2

3. (i) 50.28cm^2 (ii) 603.45cm^2 (iii) 12.57cm^2

4. (i) 154cm^2 (ii) Rs. 276 (iii) 22cm

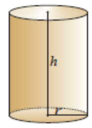
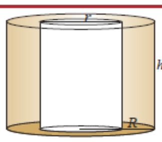
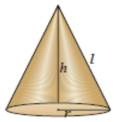
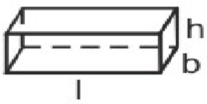
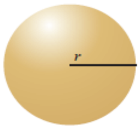
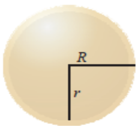
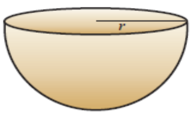
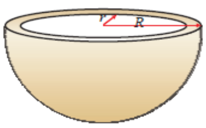
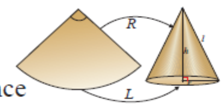
5. (i) 50 (ii) 8 (iii) 5.142 m^2

******

CHAPTER -13

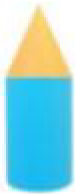
SURFACE AREAS AND VOLUMES

Prepared by, MANISHA JAISWAL
TGT (MATHS)
KV CCL RANCHI

Sl. No	Name	Figure	Lateral or Curved Surface Area (sq.units)	Total Surface Area (sq.units)	Volume (cu.units)
1	**Right circular cylinder**		$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
2	**Right circular hollow cylinder**		$2\pi h(R+r)$	$2\pi(R+r)(R-r+h)$	$\pi R^2 h - \pi r^2 h$ $\pi h(R^2-r^2)$ $\pi h(R+r)(R-r)$
3	**Right circular cone**		πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
4	Cuboid		2h(l+ b)	2(lb + bh + hl)	lbh
5	**Sphere**		$4\pi r^2$	- - -	$\frac{4}{3}\pi r^3$
6	**Hollow sphere**		- - -	- - -	$\frac{4}{3}\pi(R^3-r^3)$
7	**Hemisphere**		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
8	**Hollow Hemisphere**		$2\pi(R^2+r^2)$	$2\pi(R^2+r^2)+\pi(R^2-r^2)$	$\frac{2}{3}\pi(R^3-r^3)$
9	**Cone** CSA of a cone = Area of the sector $\pi rl = \frac{\theta}{360}\times\pi r^2$ Length of the sector = Base circumference of the cone		$l=\sqrt{h^2+r^2}$ $h=\sqrt{l^2-r^2}$ $r=\sqrt{l^2-h^2}$	10. Volume of water flows out through a pipe = {Cross section area × Speed × Time } 11. No. of new solids obtained by recasting $= \frac{\text{Volume of the solid which is melted}}{\text{volume of one solid which is made}}$	
12	Conversions	1 m³ = 1000 litres , 1 d.m³ = 1 litre , 1000 cm³ = 1 litre , 1000 litres = 1 *kl*			

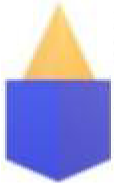
SURFACE AREAS AND VOLUMES OF COMBINATIONS OF SOLIDS

Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones



T. S. A. = C. S. A. (Cylinder) + C. S. A. (Cone) + Base Area (Cylinder)

Volume = Volume (Cone) + Volume (Cylinder)



T. S. A. = T. S. A. (Cube) + C. S. A. (Cone) − Base Area (Cone)

Volume = Volume (Cube) + Volume (Cone)

T. S. A. = T. S. A. (Cube) + T. S. A. (Cylinder) − Base Area (Cylinder)

Volume = Volume (Cube) + Volume (Cylinder)

T. S. A. = T. S. A. (Cube) + C. S. A. (Hemisphere) − Base Area (Hemisphere)

Volume = Volume (Cube) + Volume (Hemisphere)

Some solved examples

1. (*) From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and same diameter is hollowed out. Find the total surface area of the remaining solid to the nearest cm^2.

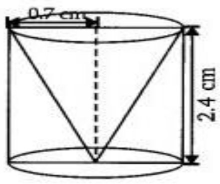
Solution:

Radius of cylinder, $r = \frac{1.4}{2} = 0.7$ cm

Height of cylinder, $h = 2.4$ cm

Radius of cone, $r = 0.7$ cm

Height of cone, $h = 2.4$ cm



Total surface area of the remaining solid

= Outer curved surface area of cylinder + Area of bottom of cylinder + Inner curved surface area of conical cavity

$$= 2\pi rh + \pi r^2 + \pi rl$$

$$= \pi r(2h + r + l)$$

$$= \frac{22}{7} \times 0.7\left[2 \times 2.4 + 0.7 + \sqrt{(0.7)^2 + (2.4)^2}\right]$$

$$= 22 \times 0.1\ [4.8 + 0.7 + 2.5]$$

$$= 22 \times 0.1 \times 8 = 17.6 \text{ cm}^2 \approx \mathbf{18\ cm^2}.$$

2.(***) A solid consisting of a right circular cone of height 120 cm and radius 60 cm standing on a hemisphere of radius 60 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is 60 cm and its height is 180 cm.

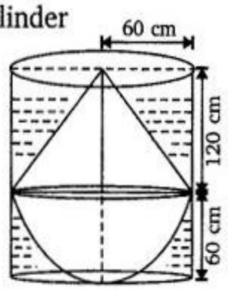
Radius of cylinder, R = 60 cm

Height of cylinder, H = 180 cm

$\therefore$ Volume of water in the cylinder

$= \pi R^2 H = \pi(60)^2 \times 180$

$= (\pi \times 3600 \times 180)\text{cm}^3$



Volume of water flows out

= Volume of conical part

+ Volume of hemispherical part

$$= \left[\frac{1}{3}\pi(60)^2 \times 120 + \frac{2}{3}\pi(60)^3\right]\text{cm}^3$$

$$= \left[\frac{4}{3}\pi(60)^3\right]\text{cm}^3$$

Volume of water left in the cylinder

$$= \left[\pi \times 3600 \times 180 - \frac{4}{3}\pi \times 3600 \times 60\right]\text{cm}^3$$

$$= \left[3600 \times 60\pi\left(3 - \frac{4}{3}\right)\right]\text{cm}^3$$

$$= \left[3600 \times 60 \times \frac{22}{7} \times \frac{5}{3}\right]\text{cm}^3$$

$= 1131428.57 \text{ cm}^3$

$= \frac{1131428.57}{1000000} \text{ m}^3 = \mathbf{1.131 \text{ m}^3}$ (approx).

MULTIPLE CHOICE QUESTIONS (MCQs)

Q1. In a right angled triangle the sides including the right angles are 3 cm and 4 cm. If the triangle is rotated about 4cm, then the volume of the solid thus generated is

a) $48\,\pi cm^3$ b) $12\,\pi cm^3$ c) $36\,\pi cm^3$ d) $16\,\pi cm^3$

Q2. The ratio of the total surface area of a solid hemisphere to the square of its radius is

a) 2 : 1   b) $3\,\pi$: 1   c) $4\,\pi$: 1   d) 1: $4\,\pi$

Q3. The radius of a wheel is 0.25 m. The number of revolutions it will make to travel a distance of 11 km is

a) 6500   b) 600   c) 7000   d) 7500

Q4. If the volume of a cube is 1331 cm^3, then the length of its edge is

a) 11 cm   b) 14 cm   c)13 cm   d) 12 cm

Q5. If the volume and the surface area of a sphere are numerically equal, then the radius of the sphere is

a) 2 units   b) 1 unit   c) 3 units   d) 4 units

Q6. A cylinder and a cone are of same base radius and of same height. The ratio of the volumes of cylinder to that of the cone is

a) 1:3   b) 2:1   c) 3:1   d) 1:2

Q7. A cylinder, a cone and a hemisphere are of same base and have the same height. The ratio of their volumes is

a) 3:1:2   b) 1:2:3   c) 2:3:1   d) 1:1:3

Q8. If radius of a sphere is $\frac{2d}{3}$ then its volume is

a) $\frac{32}{81}\pi d^3$ b) $\frac{23}{4}\pi d^3$ c) $\frac{32}{3}\pi d^3$ d) $\frac{34}{3}\pi d^3$

Q9. A surahi is the combination of

a) a sphere and a cylinder

b) a hemisphere and a cylinder

c) two hemispheres

d) a cylinder and a cone

Q10. If the diameter of the sphere is doubled, the surface area of the resultant becomes x times that of the original one. Then the value of x is

a) 2   b) 3   c) 4   d) 5

Q11. From a solid circular cylinder with height 10 cm and radius of the base 6 cm, a right circular cone of the same base and same height is removed, then the volume of the remaining solid is

a) $280\,\pi cm^3$ b) $330\,\pi cm^3$ c) $240\,\pi cm^3$ d) $440\,\pi cm^3$

Q12. The edge of a cube whose volume is equal to that of a cuboid of dimensions 8 cm x 4 cm x 2 cm is

a) 6 cm b) 4 cm c) 2 cm d) 6 cm

Q13. The sum of the length , breadth and height of a cuboid is $6\sqrt{3}$ cm and the length of its diagonal is $2\sqrt{3}$ cm . The total surface area of the cuboid is

a) 48 cm^2 b) 72 cm^2 c) 96 cm^2 d) 108 cm^2

Q14. The radius of a sphere is r cm. The sphere is divided into two equal parts. The whole surface areaof two parts will be:

a) $8\pi r^2$ b) $6\pi r^2$ c) $4\pi r^2$ d) $3\pi r^2$

Q15.The radius of a wire is decreased to one third. If the volume remains the same, the length will become

a) 3 times b) 6 times c) 9 times d) 27 times

Q16. The ratio of the volumes of two spheres is 8 : 27 . If r and R are the radii of spheres respectively, then (R – r) : r is :

a) 1 : 2 b) 1 : 3 c) 2 : 3 d) 4 : 9

Q17. The surface area of the two spheres are in the ratio 1 : 2. The ratio of their volumes is :

a) $\sqrt{2}$: 1 b) 1 : $2\sqrt{2}$ c) 1 : 8 d) 1 : 4

Q18. If the areas of three adjacent faces of a cuboid are X, Y and Z respectively, then the volume of cuboid is :

a) XYZ b) 2XYZ c) $\sqrt{XYZ}$ d) $\sqrt{2XYZ}$

Q19. Assertion: A cylinder and right circular cone are having the same base and same height the volume of cylinder is three times the volume of cone

Reason: If the radius of cylinder is doubled and height is halved the volume will be doubled

a) Both Assertion and reason are correct and reason is correct explanation for Assertion

b) Both Assertion and reason are correct but reason is not correct explanation for Assertion

c) Assertion is correct but reason is false

d) Both Assertions and reason are false

Q20. Assertion: Volume of cuboid is defined as the amount of space occupied by the walls of cuboid in three dimensional space

Reason: Volume of cuboid is the product of length ,width ,height

a) Both Assertion and reason are correct and reason is correct explanation for Assertion

b) Both Assertion and reason are correct but reason is not correct explanation for Assertion

c) Assertion is correct but reason is false

d) Both Assertions and reason are false

Q21.Assertion: The lateral surface area of a right cone is 62.82 cm^2, if the radius is 4 cm and the slant height is 5 cm.

Reason: Lateral surface area of cone = πrl

a) Both Assertion and reason are correct and reason is correct explanation for Assertion

b) Both Assertion and reason are correct but reason is not correct explanation for Assertion

c) Assertion is correct but reason is false

d) Both Assertions and reason are false

Q22. **Assertion: Savitri had to make a model of a cylindrical kaleidoscope for her science project. She wanted to use chart paper to make the curved surface of the kaleidoscope. 550cm^2 would be the area of chart paper required by her, if she wanted to make a kaleidoscope of length 25 cm with a 3.5 cm radius.**

Reason: Area of chart paper required = curved surface area of the kaleidoscope= 2πrh

a) Both Assertion and Reason are correct and reason is correct explanation for Assertion.

b) Both Assertion and Reason are false but reason is not correct explanation for Assertion.

c) Assertion is correct but reason is false.

d) Both Assertion and reason are false.

SHORT ANSWER-I (SA-I)

Q23. Find the volume of the largest right circular cone that can be cut out from a cube of edge 4.2 cm.*

Q24. The radii of two cylinders are in the ratio 3:5 and their heights are in the ratio 2:3 . What is the ratio of their curved surface areas?**

Q25. Two cubes each of volume27cm^3 are joined end to end to form a solid. Find the surface area of the solid. ***

Q26.Two cubes each of side 4cm are joined end to end. Find the volume of the resulting solid.*

Q27. Volume and surface area of a solid hemisphere are numerically equal. What is the diameter of hemisphere? *

Q28. If the total surface area of a solid hemisphere is 462 cm^2, find its radius. *

Q29. How many shots each having diameter 3 cm can be made from a cuboidal lead solid of dimensions 9 cm x 11 cm x 12 cm ? **

Q30. The surface area of a sphere is 616cm^2. Find its radius *

Q31.The base radii of 2 right circular cones of the same height are in the ratio 3:5. Find the ratio of their volumes.**

Q32. From a solid cube of side 7 cm , a conical cavity of height 7 cm and radius 3 cm is hollowed out . Find the volume of the remaining solid.***

Q33. Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 9cm.*

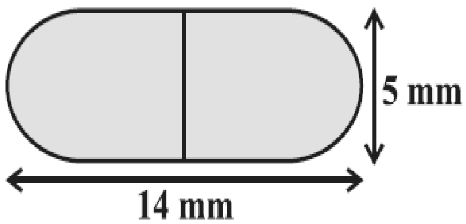
Q34. A toy is in the form of a cone mounted on a hemi-sphere of same radius. The diameter of the base of the conical part is 7cm and the total height of the toy is 14.5cm. Find the volume of the toy. ***

Q35. The rain water from a roof 22m x 20 m drain into a conical vessel having diameter of base as 2m and height 3.5 m. If the vessel is just full, then find the rainfall.*

Q36. The length of a hall is 20m and width is 16m. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height of the hall. **

SHORT ANSWER-II (SA-II)

Q37. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends. The l length of the entire capsule is 14 mm and the diameter of the capsule is 5 mm. Find its surface area.**



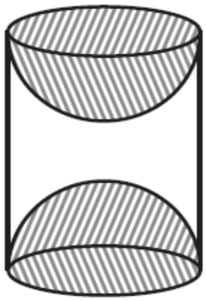
Q38. A circus tent is cylindrical up to a height of 3m and conical above it. If the diameter of the base is 105m and the slant height of the conical part is 53m. Find the total canvas required in making the tent. **

Q39. A bird-bath in a garden is in the shape of a cylinder with a hemi-spherical depression at one end. The height of the hollow cylinder is 1.45m and its radius is 30cm. Find the TSA of the bird-bath.**

Q40. A tent is in the shape of a cylinder of diameter 20m and height 2.5cm, surmounted by a cone of equal base and height 7.5m. Find the capacity of the tent.(take $\pi = 3.14$) ***

Q41. A vessel in the shape of a hollow hemi-sphere mounted by a hollow cylinder. The diameter of the hemi-sphere is 14cm and the total height of the vessel is 13cm. find the inner surface area of the vessel.**

Q42. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in the figure. If the height of the cylinder is 10 cm and its base is of radius 3.5 cm. Find the total surface of the article. ***



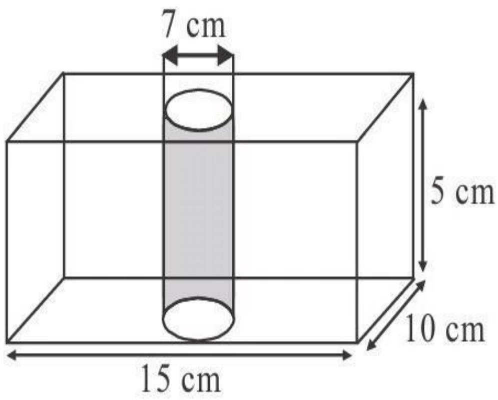
Q43. A waterhouse is used as a granary. It is in the shape of a cuboid surmounted by a half- cylinder. The base of the waterhouse is 6m x 14m and its height is 8m. Find the surface area of non-cuboidal part of the waterhouse.**

Q44. A building is in the form of a cylinder surmounted by a hemispherical vaulted dome and contains $41\frac{19}{21}$ $m^3$of air. If the internal diameter of dome is equal to its height above the floor. Find the height of the building. **

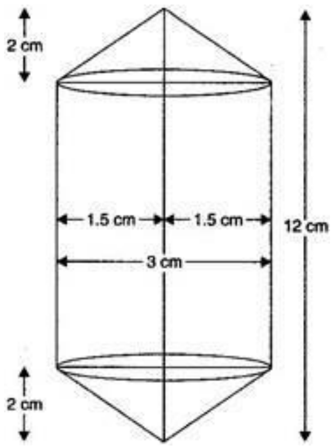
(LONG ANSWER)

Q45.(*) Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively decided to provide place and the canvas for 1500 tents and share the whole expenditure equally. The lower part of each tent is cylindrical with base radius 2.8 m and height 3.5 m and upper part is conical with the same base radius, but of height 2.1 m . If the canvas used to make the tent costs Rs.120 per m^2, find the amount shared by each school to set up the tents.

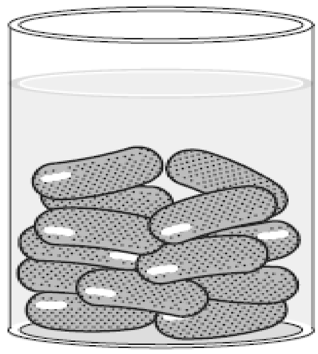
Q46. (**)From a cuboidal solid metallic block of dimensions 15cm x 10cm x 5cm a cylindrical hole of diameter 0.07m is drilled out. Find the surface area of the remaining block. (π= 22/7)



Q47.(**) Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminium sheet. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of air contained in the model that Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same).



Q48.(**) A *gulab jamun*, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 *gulab jamuns*, each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm .

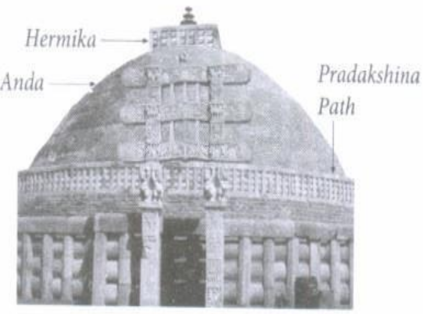


Q49.(*) If a hollow cube of internal edge 22cm is filled with spherical marbles of diameter 0.5 cm and is assumed that $\frac{1}{8}$ space of the cube remains unfilled. Then find the number of marbles that the cube can accommodate.

Q50.(**) From a solid cylinder whose height is 12 cm and diameter is 10 cm, a conical cavity of same diameter is hollowed out. Find the volume and total surface area of the remaining solid.

CASE STUDY BASED QUESTIONS

Q51. Amar is a Class X student. His class teacher Mrs Somya arranged a historical trip to great Stupa of Sanchi. She explained that Stupa of Sanchi is great example of architecture in India. Its base part is cylindrical in shape. The dome of this stupa is hemispherical in shape, known as Anda. It also contains a cubical shape part called Hermika at the top. Path around Anda is known as Pradakshina Path.



Based on the above information, answer the following questions.

(i) Find the lateral surface area of the Hermika, if the side of cubical part is 8 m. (1)

(ii) The diameter and height of the cylindrical base part are respectively 42 m and 12 m. If the volume of each brick used is 0.01 m3, then find the number of bricks used to make the cylindrical base. (2)

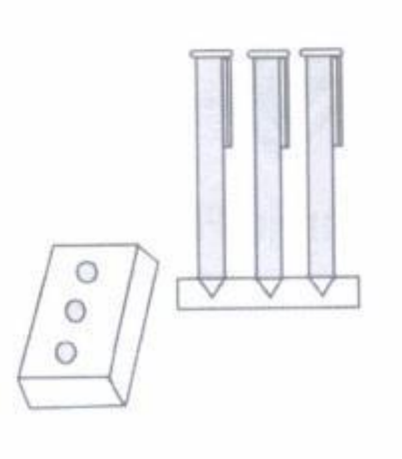
(iii) Find the Curverd surface area of Anda if its radius is 21m. (1)

Q52. A carpenter used to make and sell different kinds of wooden pen stands like rectangular, cuboidal, cylindrical, conical. Aanav went to his shop and asked him to make a pen stand as explained below. Pen stand must be of the cuboidal shape with three conical depressions, which can hold 3 pens. The dimensions of the cuboidal part must be 20 cm x 15 cm x 5 cm and the radius and depth of each conical depression must be 0.6 cm and 2.1cm respectively

i) What will be the volume of cuboidal part? (1)

ii) What is the total volume of conical depression? (1)

iii) Find the total cost of making the pen stand,if the cost of wood used is 0.05 per cm^3. (2)



Q53. Alok and his family went for a vacation to Jaipur. There they had a stay in tent for a night. Alok found that the tent in which they stayed is in the form of a cone surmounted on a cylinder. The total height of the tent is 42 m, diameter of the base is 42m and height of the cylinder is 22 m.

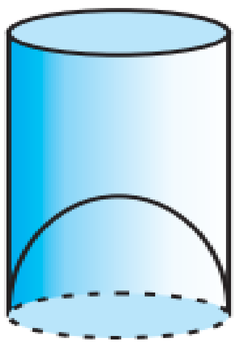
Based on the above information, answer the following questions:

i) How much canvas is needed to make the tent?(1)

ii) If each person needs 126 $m^2$of floor, then how many person can accommodated in the tent?(1)

iii) Find the number of persons that can be accommodated in tent , if each person needs 1892 m^3 of space.(2)

Q54. A juice seller was serving his customers using glasses as shown in Fig. The inner diameter of the cylindrical glass was 5 cm, but the bottom of the glass had a hemispherical raised portion which reduced the capacity of the glass. If the height of a glass was 10 cm, find the apparent capacity of the glass and its actual capacity. (Use π = 3.14.)



i) Find the apparent capacity of the glass. (1)

ii) Find the actual capacity of the glass. (2)

iii) What is the difference between apparent and actual capacity of the glass (1)

Q55. An antique box and its dimensions excluding the stand is given

i) What is the volume of the jewellery box ? (1)

ii) How much brass will be needed to plate the curved surface of the dome as shown in figure? (2)

iii) How many sheets of dimensions 14cm x 30cm x 2cm can be placed in the box? (1)

CHAPTER – 13 ANSWER KEY

Q.NO.	ANSWER	Q.NO.	ANSWER
1	b) 12π cm^3	31	2:5
2	b)3 π:1	32	277 cm^3
3	c) 7000	33	462 cm^3
4	a) 11 cm	34	190.93 cm^3
5	d) 4 units	35	0.83 cm
6	c) 3:1	36	3.5 cm, 7 cm
7	a)3:1:2	37	220 mm^2
8	a)$\frac{32}{81}\pi$d^3	38	28 cm
9	a) A sphere and a cylinder	39	9735 m^2
10	C) 4	40	3.3 m^2
11	c)240 π cm^3	41	1570 m^2
12	C)2 cm	42	374 cm^2
13	C)96 cm^2	43	$\frac{1122}{7}$ m^2
14	b) 6 πr^2	44	4m
15	c) 9 times	45	Rs. 3,32,640
16	a) 1:2	46	583 cm^2
17	b)1:2$\sqrt{2}$	47	66 cm^3
18	c)$\sqrt{xyz}$	48	338 cm^3
19	b	49	142296
20	b	50	628.57 cm^3/ 660 cm^2
21	a	51	i)256 m^2 ii) 16,63,200 bricks iii)882π cm^2
22	a	52	i)1500 cm^3 ii)2.376 cm^3 iii)Rs. 748.80
23	19.404^3	53	i)4818 m^2 ii)11 iii)21
24	2:5	54	i)196.25 cm^3 ii)163.54 cm^3 iii)32.71 cm^3
25	90 cm^2	55	i)lbh+$\frac{1}{2}\pi$r^{2}h ii) 660 cm^2 iii)5
26	128 cm^3		
27	9 units		
28	7 cm		
29	84		
30	7cm		

******

CHAPTER -14

STATISTICS

Prepared by, Puja Kumari
TGT (MATHS)
KV Patratu

- **Statistics** is one of the parts of mathematics in which we study about the collecting, organizing, analyzing, interpreting and presenting data
- **Ungrouped data** - Ungrouped data is data in its original or raw form. The observations are not classified into groups.
- **Grouped data** - In grouped data, observations are organized in groups.

For example, a class of students got different marks in periodic test. The data is tabulated as below

Marks interval	0-10	10-20	20-30	30-40
No. of students	4	8	12	16

- **Frequency** (f) -Frequency is the number of times a particular observation occurs in data.
- **Class Interval** - Data can be grouped into class intervals such that all observations in that range belong to that class.
- **Class width/Class Size** (h) = upper class limit – lower class limit

Three measures of central tendency

A) Mean ($\overline{x}$) **B) Median** **C) Mode**

A) METHODS OF FINDING MEAN

i) Direct Method:

$$\overline{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

x_i = Class mark

$$\text{Class mark} = \frac{\text{Upper class limit} + \text{Lower class limit}}{2}$$

f_i = frequency

Example: Find Mean by Direct method:

Class interval	10 - 25	25 - 40	40 - 55	55 - 70	70 - 85	85 - 100
Number of students	2	3	7	6	6	6

Solution:

Class interval	Number of students (f_i)	Class mark (x_i)	$f_i x_i$
10 - 25	2	17.5	35.0
25 - 40	3	32.5	97.5
40 - 55	7	47.5	332.5
55 - 70	6	62.5	375.0
70 - 85	6	77.5	465.0
85 - 100	6	92.5	555.0
Total	$\Sigma f_i = 30$		$\Sigma f_i x_i = 1860.0$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{1860.0}{30} = 62$$

This method of finding the mean is known as the **Direct Method.**

ii) Assumed Mean Method

$$\textbf{Mean } (\bar{x}) = a + \frac{\sum_{i=1}^{n} \text{fidi}}{\sum_{i=1}^{n} \text{fi}}$$

where a = assumed mean

$d_i = x_i - a$

Example: Find Mean by Assumed Mean method:

Class interval	10 - 25	25 - 40	40 - 55	55 - 70	70 - 85	85 - 100
Number of students	2	3	7	6	6	6

Class interval	Number of students (f_i)	Class mark (x_i)	$d_i = x_i - 47.5$	$f_i d_i$
10 - 25	2	17.5	–30	–60
25 - 40	3	32.5	–15	–45
40 - 55	7	47.5	0	0
55 - 70	6	62.5	15	90
70 - 85	6	77.5	30	180
85 - 100	6	92.5	45	270
Total	$\Sigma f_i = 30$			$\Sigma f_i d_i = 435$

the mean of the deviations, $\bar{d} = \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

Mean of deviations, $\bar{d} = \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

So,
$$\bar{d} = \frac{\Sigma f_i (x_i - a)}{\Sigma f_i}$$
$$= \frac{\Sigma f_i x_i}{\Sigma f_i} - \frac{\Sigma f_i a}{\Sigma f_i}$$
$$= \bar{x} - a\frac{\Sigma f_i}{\Sigma f_i}$$
$$= \bar{x} - a$$

So, $\bar{x} = a + \bar{d}$

i.e., $\bar{x} = a + \dfrac{\Sigma f_i d_i}{\Sigma f_i}$

Substituting the values of a, $\Sigma f_i d_i$ and Σf_i from Table 14.4, we get

$$\bar{x} = 47.5 + \frac{435}{30} = 47.5 + 14.5 = 62.$$

Therefore, the mean of the marks obtained by the students is 62.

The method discussed above is called the **Assumed Mean Method**.

B) Mode

Mode of grouped data can be found as

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

where l = lower limit of the modal class,

h = size of the class interval (assuming all class sizes to be equal),

f_1 = frequency of the modal class,

f_0 = frequency of the class preceding the modal class,

f_2 = frequency of the class succeeding the modal class.

Example:

A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household:

Family size	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11
Number of families	7	8	2	2	1

Find the mode of this data.

Solution :

The maximum class frequency is 8, and the class corresponding to this frequency is 3 – 5. So, the modal class is 3 – 5.

Now modal class = 3 – 5,

lower limit (l) of modal class = 3,

class size (h) = 2

frequency (f_1) of the modal class = 8,

frequency (f_0) of class preceding the modal class = 7,

frequency (f_2) of class succeeding the modal class = 2.

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 3 + \left(\frac{8 - 7}{2 \times 8 - 7 - 2}\right) \times 2 = 3 + \frac{2}{7} = 3.286$$

Therefore, the mode of the data above is 3.286.

C) Median

The median for grouped data can be found by using the formula

$$\text{Median} = l + \frac{\frac{n}{2} - c.f.}{f} \times h$$

Where l = lower limit of the median class

n = number of observations

c.f. = cumulative frequency of the class preceding the median class

h = class width

Example:

The following table gives the distribution of the life time of 400 neon lamps :

Life time (in hours)	Number of lamps
1500 - 2000	14
2000 - 2500	56
2500 - 3000	60
3000 - 3500	86
3500 - 4000	74
4000 - 4500	62
4500 - 5000	48

Find the median life time of a lamp

Solution:

Lifetime (in hours)	Number of lamps	*cf*
1500 – 2000	14	14
2000 – 2500	56	70
2500 – 3000	60	130
3000 – 3500	86	216
3500 – 4000	74	290
4000 – 4500	62	352
4500 – 5000	48	400
Total	400	

Here, $\frac{n}{2} = \frac{400}{2} = 200$

$\therefore$ Median class = 3000 – 3500

So, $f = 86, cf = 130, h = 500$

We have, $\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$$= 3000 + \left(\frac{200 - 130}{86}\right) \times 500$$

$$= 3000 + \frac{35000}{86} = 3000 + 406.98 = 3406.98 \text{ hours}$$

- **There is a empirical relationship between the three measures of central tendency:**

$$\boxed{\textbf{3 Median = Mode + 2 Mean}}$$

Practice Questions

I. Multiple Choice Questions (1marks each)

*i) If the mode of a distribution is 8 and its mean is also 8, then its median is

a) 10 b) 8 c) 7 d) 6

ii)Consider the following distribution:

Marks obtained	0 or more	10 or more	20 or more	30 or more	40 or more	50 or more
No. of students	63	58	55	51	48	42

The frequency of the class 30-40 is

a) 3 b) 4 c) 48 d) 5

**iii) Consider the following frequency distribution of the heights of 60 students of a class

Height in cm	150-155	155-160	160-165	165-170	170-175	175-180
No. of students	15	13	10	8	9	5

The sum of the lower limit of the modal class and upper limit of the median class is.

(a) 310 (b) 315 (c) 320 (d) 330

iv)The modal class of the following distribution is.

class	0-10	10-20	20-30	30-40	40-50	50-60
frequency	3	9	15	30	18	5

a) 40-50 b)20-30 c) 30-40 d) 50-60

v) The median class of the following frequency distribution is

class	0-10	10-20	20-30	30-40	40-50	50-60
frequency	8	10	12	22	30	18

a) 20-30 b)30-40 c) 40-50 d) none of these

*vi) For the following distribution

Class	10-15	15-20	20-25	25-30	30-35
frequency	25	30	27	35	21

The sum of the lower limit of the median class and the lower limit of the modal class is

a) 45 b) 50 c) 55 d) 62

*vii)The times, in seconds, taken by 50 athletes to run a 110 m hurdle race are tabulated below.

Time in seconds	13.8-14	14-14.2	14.2-14.4	14.4-14.6
No. of athletes	2	14	16	18

The number of athletes who completed the race in less than 14.4 seconds is :

a) 2 b)32 c) 16 d)50

***viii) Relationship between mean median and mode is

a)3 Median = 2 Mode + Mean b) 3Mode = 3 Mean + Median

c) 3Median = 2Mean +Mode d) 3Mean = 2Mode + Median

ix) Consider the following frequency distribution

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	12	10	15	8	11

The median class is

a) 6-12 b) 12-18 c)18-24 d) 24-30

**x) If the mean of the following distribution is 2.6, then the value of y is

Variable (x)	1	2	3	4	5
Frequency(f)	4	5	Y	1	2

a) 3 b) 13 c) 24 d) 8

II.VERY SHORT ANSWER TYPE QUESTIONS (2 marks each)

1. Calculate the median from the following data

Marks	0-10	10-20	20-30	30-40	40-50
No. of students	5	15	30	8	2

2.Find the mode of the following frequency distribution

Class interval	0-6	6-12	12-18	18-24	24-30
Frequency	7	5	10	12	6

**3. Find the value of p, if the arithmetic mean of the following distribution is 25:

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	5	8	15	p	6

4. Find median class of the following distribution

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
frequency	4	4	8	10	12	8	4

***5. Find x and y from the following frequency distribution

Class	frequency	Cumulative frequency
0-8	15	15
8-16	X	28
16-24	15	43
24-32	18	Y
32-40	9	70

6. Find mean of the following distribution

class	3-5	5-7	7-9	9-11	11-13
frequency	5	10	10	7	8

*7. For the following distribution find the modal class

Marks	Below 10	Below 20	Below 30	Below 40	Below 50	Below 60
No of students	3	12	27	57	75	80

**8. Find the sum of lower limit of median class and upper limit of modal class

class	10-20	20-30	30-40	40-50	50-60	60-70
frequency	1	3	5	9	7	3

III. SHORT ANSWER TYPE QUESTIONS (3 marks each)

**1. The length of 40 leaves of a plant are measured correct to nearest millimetre, and the data obtained is represented in the following table.

Length (in mm)	118-126	127-135	136-144	145-153	154-162	163-171	172-180
No of leaves	3	5	9	12	5	4	9

Find the average length of the leaves

2. Find mean of the following distribution

class	20-30	30-40	40-50	50-60	60-70
frequency	25	40	42	43	10

3.The following table gives the number of participants in a yoga camp

Age (in years)	20-30	30-40	40-50	50-60	60-70
No of participants	8	40	58	90	83

Find modal age of the participants.

*4.The marks obtained by 110 students in an examination are given below

Class	30-35	35-40	40-45	45-50	50-55	55-60	60-65
frequency	14	16	28	23	18	8	3

Find the mean marks of the students.

**5.If the mean of the following frequency distribution is 18. Find the missing frequency

Class	11-13	13-15	15-17	17-19	19-21	21-23	23-25
frequency	3	6	9	13	-------	5	4

**6.If the mean of the following data is 14.7, find the values of p and q

Class	0-6	6-12	12-18	18-24	24-30	30-36	36-42	Total
frequency	10	P	4	7	Q	4	1	40

IV. Long Answer Type Questions (5 marks each)

***1. 250 apples of a box were weighed and distribution of the masses of the apples is given below in the following table.

Mass (in grams)	80-100	100-120	120-140	140-160	160-180
Number of apples	20	60	70	X	60

a) find the value of x and the mean mass of the apples

b) find the modal mass of apples

**2. The mode of the following frequency distribution is 55.Find the missing frequencies ‘a’ and ‘b’

Class interval	0-15	15-30	30-45	45-60	60-75	75-90	Total
frequency	6	7	a	15	10	b	51

***3.The median of the following data is 50. Find the values of ‘p’ and ‘q’, if the sum of all frequencies is 90. Also find the mode of the data

Marks obtained	20-30	30-40	40-50	50-60	60-70	70-80	80-90
No of students	P	15	25	20	q	8	10

***4.The following table gives the distribution of the life time of 400 neon lamps:

Lifetime (in hours)	1500-2000	2000-2500	2500-3000	3000-3500	3500-4000	4000-4500	4500-5000
No of lamps	14	56	60	86	74	62	48

Find the average lifetime of a lamp

*5.The following distribution shows the daily pocket allowance of children of a locality. The mean pocket allowance is 18. Find the missing frequency.

Daily pocket allowance (in Rs)	11-13	13-15	15-17	17-19	19-21	21-23	23-25
No of children	7	6	9	13	f	5	4

***6. The median of the following data is 525. Find the values of x and y, if the total frequency is 100.

Class interval	0-100	100-200	200-300	300-400	400-500	500-600	600-700	700-800	800-900	900-1000
frequency	2	5	x	12	17	20	y	9	7	4

**7. A life insurance agent found the following data for distribution of ages of 100 policyholders. Calculate the median age, if policies are given only to persons having age 18years onwards but less than 60 years.

Age (in years)	Below 20	20-25	25-30	30-35	35-40	40-45	45-50	50-55	55-60
No of policy holders	2	5	x	12	17	20	y	9	7

***8. The mean of the following distribution is 18. Find frequency of the class 19-21.

Class interval	11-13	13-15	15-17	17-19	19-21	21-23	23-25
Frequency	3	6	9	13	f	5	4

**9. The distribution given below shows the number of wickets taken by bowlers in one day cricket matches. Find the mean and median of the number of wickets taken.

No. of wickets	20-60	60-100	100-140	140-180	180-220	220-260
No. of bowlers	7	5	16	12	2	3

V. CASE STUDY BASED QUESTIONS

*1.A group of students decided to make a project on Statistics. They are collecting the heights (in cm) of their 51 girls of Class X A, B and C of their school. After collecting the data, they arranged the data in the following less than cumulative frequency distribution table form.

Height (in cm)	Number of girls
Less than 140	4
Less than 145	11
Less than 150	29
Less than 155	40
Less than 160	46
Less than 165	51

Class intervals	Frequency	Cumulative frequency
Below 140	4	4
140 - 145	7	11
145 - 150	18	29
150 - 155	11	40
155 - 160	6	46
160 - 165	5	51

i) What is the lower limit of the median class?

a)145 b)150 c)155 d)160 145

ii) What is the upper limit of the modal class?

a)145 b)150 c)155 d)160 150

iii) What is the mean of the lower limits of the median and modal class?

a)145 b)150 c)155 d)160 145

iv) What is the width of the class ?

a)10 b)15 c) 5 d) none of these

**2. Overweight and obesity may increase the risk of many health problems including diabetes, heart disease and certain cancers. The basic reason behind is the laziness, eating more junk foods and less physical exercise. The school management give instructions to the school to collect the weight data of each student. During medical check of 35 students from class X A, their weight was recorded as follows:

Weight (in Kg)	No of students
Less than 38	0
Less than 40	3
Less than 42	5
Less than 44	9
Less than 46	14
Less than 48	28
Less than 50	32
Less than 52	35

i) Find the median class of the above data?

a) 44-46 b) 46-48 c) 48-50 d) 0-52

ii)What is the median weight of the data?

a) 46 b) 46.5 c) 47 d) 47.5

iii) what is the mean of the above data?

a) 45.8 b) 46.2 c) 45.2 d) 46.5

iv) How many students have weight in the range of 44-46 kg ?

a) 2 b) 3 c) 5 d) 5

***3. A group of students went to another city to collect the data of monthly consumptions(in units) to complete their Statistics project. They prepare the following frequency distribution table from the collected data which gives monthly consumers of a locality.

Monthly consumption (in units)	No. of consumers
65-85	4
85-105	5
105-125	13
125-145	20
145-165	14
165-185	8
185-205	4

i) What is the lower limit of the median class?

a) 145 b) 165 c)105 d)125

ii)What is the lower limit of the modal class?

a) 105 b) 125 c)145 d) 165

iii)What is the width of the class?

a) 5 b) 10 c) 25 d) 20

iv) How many consumers' monthly consumption is more than 145 units?

a) 22 b) 14 c) 26 d) 8

***4. The COVID-19 pandemic, also known as coronavirus pandemic, is an ongoing pandemic of coronavirus disease caused by the transmission of severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) among humans.

The following tables shows the age distribution of case admitted during a day in two different hospitals.

Table 1

Age (in years)	5-15	15-25	25-35	35-45	45-55	55-65
No. of cases	6	11	21	23	14	5

Table 2

Age(in years)	5-15	15-25	25-35	35-45	45-55	55-65
No. of cases	8	16	10	42	24	12

Based on the above information answer the following questions.

i)The average age for which maximum cases occurred is (refer to table 1)

a)32.24 b)34.36 c)36.82 d)42.24

ii)The upper limit of the modal class is (refer to table 1)

a)15 b)25 c)35 d)45

iii)The mode of the given data is(refer to table2)

a)41.4 b)48.2 c)55.3 d)64.6

iv)The median of the given data is (refer to table 2)

a)32.7 b)40.2 c)42.3 d)48.6

CHAPTER 14 ANSWER KEY I MCQ

i).b	ii).a	iii)b	iv) c	v) b	vi) a	vii) b	viii) c	ix) b	x) d

II SHORT ANSWER TYPE QUESTIONS (2 marks each)

1) Median= 23.3 2) Mode =19.5 3) p=6

4. 30-40 5. x = 13 , y =61 6. Mean 8.15 7. 30-40 8. 90

III SHORT ANSWER TYPE QUESTION (3 marks each)

1) 146.75 2) Approx. 42.5 3)58.205 4)44.81 5)8

6) p=11, q=3

IV LONG ANSWER TYPE QUESTIONS (5 marks each)

1) x=40, Mean=134.8, Median132.85 2) a= 5, b=8 3) p =5, q=7, Mode= 46.67

4) 34105) f=20 6) x=9, y=15

7) 35.76 8) f=8 9) Median= 126.25; Mean= 125.33

V.CASE STUDY BASED QUESTIONS

1.

i)a	ii)b	iii)a	iv)c

2.

i) b	ii)b	iii)a	iv)c

3.

i) d	ii) b	iii) d	iv) c

4.

i) c	ii) d	iii) a	iv) b

CHAPTER -15

PROBABILITY

Prepared by, SONALI SINGH
TGT (MATHS)
KV RAMGARH CANTT

I) IMPORTANT NOTES OF PROBABILITY

Probability: It is the numerical measurement of the degree of certainty.

Formula used to find probability:

$$P(E) = \frac{\text{Number of Outcomes Favourable to E}}{\text{Number of all possible outcomes of the experiment}}$$

- If P(E) = 1, then it is called a 'Certain Event'.
- If P(E) = 0, then it is called an 'Impossible Event'.
- The probability of an event E is a number P(E) such that: $0 \le P(E) \le 1$
- An event having only one outcome is called an elementary event. The sum of the probabilities of all the elementary events of an experiment is 1.
- For any event E, P(E) + P(*NOT E*) = 1, E and *[NOT E]* is called complementary events.

Sum of Probabilities

The **sum** of the probabilities of all the **elementary events** of an experiment is **one**.

Example: take the coin-tossing experiment

P(Heads) + P(Tails)

= (1/2)+ (1/2) =1

Impossible Event:

An event that has **no chance of occurring** is called an **Impossible event**,

i.e. P(E) = 0.

E.g., The probability of getting a 7 on a roll of a die is 0. As 7 can never be an outcome of this trial.

Sure event:

An event that has a **100% probability** of occurrence is called a **sure event**.

E.g., What is the probability that a number obtained after throwing a die is less than 7?
So, P(E) = P(Getting a number less than 7) = 6/6= 1

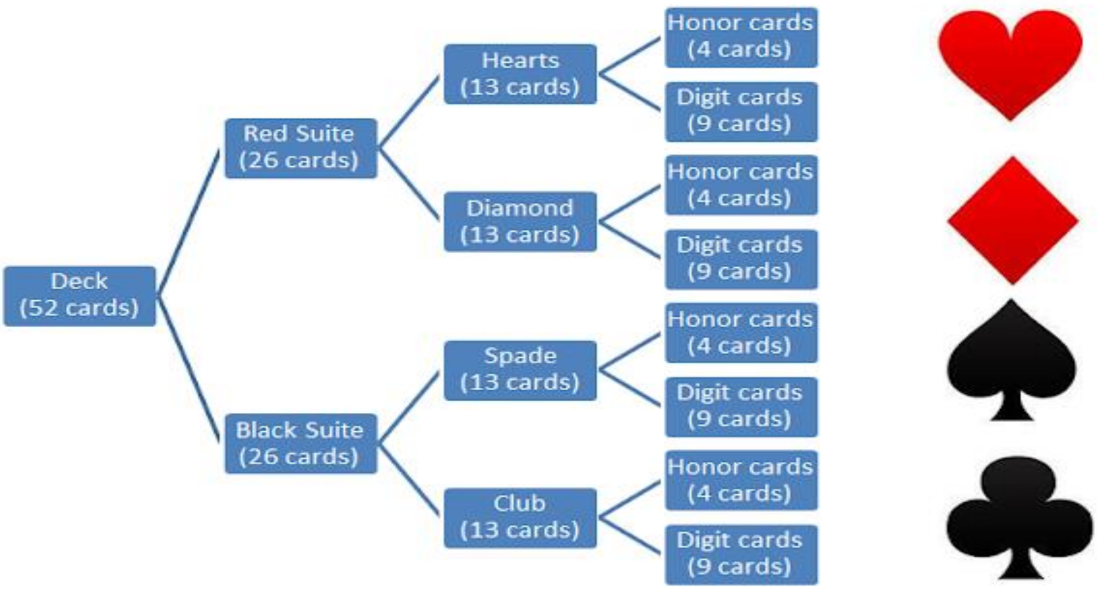
Range of Probability of an event:

Probability can range between 0 and 1, where 0 probability means the event to be an impossible one and probability of 1 indicates a certain event

i.e. $0 \leq P(E) \leq 1$.

Important POINTS

- **Coin:** A coin has two faces termed a Head and Tail.**If we toss a coin, it would be HEAD or TAIL,only two outcomes.**
- **Dice:** A dice is a small cube that has between one to six spots or numbers on its sides, which is used in games.**A DICE has six outcomes numbering 1 to 6.**
- **Cards:** A pack of playing cards(52CARDS) consists of four suits called Hearts, Spades, Diamonds, and Clubs. Each suite consists of 13 cards.



II) EXAMPLES RELATED TO PROBABILITY

Example 1: A bag contains only lemon-flavoured candies. Arjun takes out one candy without looking into the bag. What is the probability that he takes out an orange-flavoured candy?

Solution: Let us take the number of candies in the bag to be 100.

Number of orange-flavoured candies = 0 [since the bag contains only lemon-flavoured candies]

Hence, the probability that he takes out an orange-flavoured candy is:

P (Taking orange-flavoured candy) = Number of orange-flavoured candies / Total number of candies.= 0/100 = 0

Hence, the probability that Arjun takes out an orange-flavoured candy is 0.

This proves that the probability of an impossible event is 0.

Example 2: **A game of chance consists of spinning an arrow that comes to rest, pointing at any one of the numbers such as 1, 2, 3, 4, 5, 6, 7, or 8, and these are equally likely outcomes. What is the probability that it will point at? (i) 8 (ii) Number greater than 2 (iii) Odd numbers**

Solution: Sample Space = {1, 2, 3, 4, 5, 6, 7, 8}

Total Numbers = 8

(i) Probability that the arrow will point at 8:

Number of times we can get 8 = 1

P (Getting 8) = 1/8.

(ii) Probability that the arrow will point at a number greater than 2:

Number greater than 2 = 3, 4, 5, 6, 7, 8.

No. of numbers greater than 2 = 6

P (Getting numbers greater than 2) = 6/8 = 3/4.

(iii) Probability that the arrow will point at the odd numbers:

Odd number of outcomes = 1, 3, 5, 7

Number of odd numbers = 4.

P (Getting odd numbers) = 4/8 = ½.

EXAMPLE 3: One card is drawn from a deck of 52 cards, well-shuffled. Calculate the probability that the card will(i) be an ace, (ii) not be an ace.

Solution: Well-shuffling ensures equally likely outcomes.

(i) There are 4 aces in a deck.Let E be the event the card drawn is ace.

The number of favourable outcomes to the event E = 4

The number of possible outcomes = 52

Therefore, P(E) = 4/52 = 1/13

(ii) Let F is the event of 'card is not an ace'

The number of favourable outcomes to F = 52 – 4 = 48

The number of possible outcomes = 52

Therefore, P(F) = 48/52 = 12/13

III) SOLVED PRACTICE SET

SECTION A - MCQ QUESTIONS:1 MARKS

*1. The probability of event equal to zero is called;

(a) Unsure event (b) Sure Event (c) Impossible event (d) Independent event

Answer: (c) Impossible event

**2. The probability that cannot exist among the following:

(a) ⅔ (b) -1.5 (c) 15% (d) 0.7

Answer: (b) -1.5

***3. If P(E) = 0.07, then what is the probability of 'not E'?

(a) 0.93 (b) 0.95 (c) 0.89 (d) 0.90

Answer: (a) 0.93

**4. A bag has 3 red balls and 5 green balls. If we take a ball from the bag, then what is the probability of getting red balls only?

(a) 3 (b) 8 (c) 3/8 (d) 8/3

Answer: (c) 3/8

5. A bag has 5 white marbles, 8 red marbles and 4 purple marbles. If we take a marble randomly, then what is the probability of not getting purple marble?

(a) 0.5 (b) 0.66 (c) 0.08 (d) 0.77

Answer: (d) 0.77

***6. A dice is thrown in the air. The probability of getting odd numbers is

(a) ½ (b) 3/2 (c) 3 (d) 4

Answer: (a) ½

*7. If we throw two coins in the air, then the probability of getting both tails will be:

(a) ½ (b) ¼ (c) 2 (d) 4

Answer: (b) ¼

8. If two dice are thrown in the air, the probability of getting sum as 3 will be

(a) 2/18 (b) 3/18 (c) 1/18 (d) 1/36

Answer: (c) 1/18

***9. A card is drawn from the set of 52 cards. Find the probability of getting a queen card.

(a) 1/26 (b) 1/13 (c) 4/53 (d) 4/13

Answer: (b) 1/13

10. A fish tank has 5 male fish and 8 female fish. The probability of fish taken out is a male fish:

(a) 5/8 (b) 5/13 (c) 13/5 (d) 5

Answer: (b) 5/13

SECTION B : SHORT ANSWER I (SA I): 2 MARK QUESTIONS

***Q. 1: Two dice are thrown at the same time. Find the probability of getting**

(i) the same number on both dice.

(ii) different numbers on both dice.

Solution: Given that, Two dice are thrown at the same time.

So, the total number of possible outcomes n(S) = 6^2 = 36

(i) Getting the same number on both dice:

Let A be the event of getting the same number on both dice.

Possible outcomes are (1,1), (2,2), (3, 3), (4, 4), (5, 5) and (6, 6).

Number of possible outcomes = n(A) = 6

Hence, the required probability =P(A) = n(A)/n(S)

= 6/36= 1/6

(ii) Getting a different number on both dice.

Let B be the event of getting a different number on both dice.

Number of possible outcomes n(B) = 36 – Number of possible outcomes for the same number on both dice

= 36 – 6 = 30

Hence, the required probability = P(B) = n(B)/n(S)

= 30/36

= 5/6

****Q. 2: A bag contains a red ball, a blue ball and a yellow ball, all the balls being of the same size. Kritika takes out a ball from the bag without looking into it. What is the probability that she takes out the(i) yellow ball?(ii) red ball?**

Solution: Kritika takes out a ball from the bag without looking into it. So, it is equally likely that she takes out any one of them from the bag.

Let Y be the event ‘the ball taken out is yellow’, B be the event ‘the ball taken out is blue’, and R be the event ‘the ball taken out is red’.

The number of possible outcomes = Number of balls in the bag = n(S) = 3.

(i) The number of outcomes favourable to the event Y = n(Y) = 1.

So, P(Y) = n(Y)/n(S) =1/3

Similarly, (ii) P(R) = 1/3

****Q.3: One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will(i) be an ace,(ii) not be an ace.**

Solution: Well-shuffling ensures equally likely outcomes.

(i) Card drawn is an ace

There are 4 aces in a deck.

Let E be the event ‘the card is an ace’.

The number of outcomes favourable to E = n(E) = 4

The number of possible outcomes = Total number of cards = n(S) = 52

Therefore, P(E) = n(E)/n(S) = 4/52 = 1/13

(ii) Card drawn is not an ace

Let F be the event ‘card drawn is not an ace’.

The number of outcomes favourable to the event F = n(F) = 52 – 4 = 48

Therefore, P(F) = n(F)/n(S) = 48/52 = 12/13

***Q.4: Two dice are numbered 1, 2, 3, 4, 5, 6 and 1, 1, 2, 2, 3, 3, respectively. They are thrown, and the sum of the numbers on them is noted. Find the probability of getting each sum 2 and 9 separately.**

Solution: Number of total outcome = n(S) = 36

(i) Let E_1 be the event 'getting sum 2'

Favourable outcomes for the event E_1 = {(1,1),(1,1)}

$n(E_1) = 2$

P(E1) = n(E1)/n(S) = 2/36 = 1/18

(ii) Let E_8 be the event 'getting sum 9'

Favourable outcomes for the event E_8 = {(6,3),(6,3)}, $n(E_8) = 2$

$P(E_8) = n(E_8)/n(S) = 2/36 = 1/18$

*****Q. 5:** i) **If P(E) = 0.05, what is the probability of 'not E'?**

ii) Write the probability of an impossible event.

Solution:i) We know that,P(E) + P(not E) = 1

It is given that, P(E) = 0.05So, P(not E) = 1 – P(E)

P(not E) = 1 – 0.05 =>P(not E) = 0.95

ii) P(E) = 0 , FOR an impossible event

<u>SECTION C : SHORT ANSWER II(SA II): 3 MARKS QUESTIONS</u>

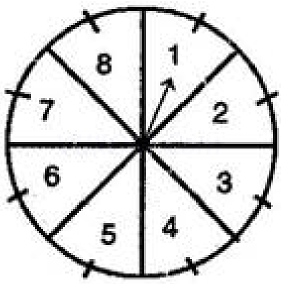
****1. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see figure) and these are equally likely outcomes. What is the probability that it will point at: (any 3)**

(i) 8?

(ii) an odd number?

(iii) a number greater than 2?

(iv) a number less than 9?



Ans. Out of 8 numbers, an arrow can point any of the numbers in 8 ways.

$\therefore$ Total number of favourable outcomes = 8

(i) Favourable number of outcomes = 1

Hence, P (arrow points at 8) = $\frac{1}{8}$

(ii) Favourable number of outcomes = 4

Hence, P (arrow points at an odd number) = $\frac{4}{8} = \frac{1}{2}$

(iii) Favourable number of outcomes = 6

Hence, P (arrow points at a number > 2) = $\frac{6}{8} = \frac{3}{4}$

(iv) Favourable number of outcomes = 8

Hence, P (arrow points at a number < 9) = $\frac{8}{8}$ =1

*****2. A dice is thrown once. Find the probability of getting:**

(i) a prime number.

(ii) a number lying between 2 and 6.

(iii) an odd number.

Ans. Total number of favourable outcomes of throwing a dice = 6

(i) On a dice, the prime numbers are 2, 3 and 5.

Therefore, favourable outcomes = 3

Hence P (getting a prime number) = $\frac{3}{6} = \frac{1}{2}$

(ii) On a dice, the number lying between 2 and 6 are 3, 4, 5.

Therefore, favourable outcomes = 3

Hence P (getting a number lying between 2 and 6) = $\frac{3}{6} = \frac{1}{2}$

(iii) On a dice, the odd numbers are 1, 3 and 5.

Therefore, favourable outcomes = 3

Hence P (getting an odd number) = $\frac{3}{6} = \frac{1}{2}$

***3. A game consists of tossing a one rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e., three heads or three tails and loses otherwise. Calculate the probability that Hanif will lose the game.**

Ans. The outcomes associated with the experiment in which a coin is tossed thrice:

HHH, HHT, HTH, THH, TTH, HTT, THT, TTT

Therefore, Total number of favourable outcomes = 8

Number of favourable outcomes = 6

Hence required probability = $\frac{6}{8} = \frac{3}{4}$

****4. 18 cards numbered 1, 2, 3, …18 are put in a box and mixed thoroughly. A card is drawn at random from the box. Find the probabilities that the card bears**

(i) an even number

(ii) a number divisible by 2 or 3

Ans. Total no. of possible outcomes = 18

(i) Favorable cases are 2,4,6,8,10,12,14,16,18 i.e., 9 in number

Required probability = $\frac{9}{18} = \frac{1}{2}$

(ii) Favorable cares are 2,3,4,6,8,9,10,12,14,15,16,18 i.e., 12 in number

Required probability = $\frac{12}{18} = \frac{2}{3}$

*****5. A bag contains 5 red balls, 4 green balls and 7 white balls. A ball is drawn at random from the box. Find the probability that the ball drown is**

(a) white

(b) neither red nor white

Ans . Total number of balls in the bag = 5+4+7=16

Total number of possible outcomes = 16

(a) Favourable outcomes for a white ball = 7

Required probability = $\frac{7}{16}$

(b) Favourable outcomes for neither red nor white ball=Number of green balls =4

∴ Required probability = $\frac{4}{16} = \frac{1}{4}$

****6. A box contains 20 balls bearing numbers 1,2,3,4,…20. A ball is drawn at random from the box, what is the probability that the number on the ball is**

(i) an odd number

(ii) divisible by 2 or 3

(iii) prime number

Ans. Total number of outcomes = 20

(i) Favorable outcomes are 1,3,5,7,9,11,13,15,17,19 i.e., 10 in number.

∴ Required probability = $\frac{10}{20} = \frac{1}{2}$

(ii) Number "divisible by 2" are 2,4,6,8,10,12,14,16,18,20 i.e., 10 in number

Numbers "divisible by 3 are 3,6,9,12,15,18. i.e., 6 in number

Numbers "divisible by 2 or 3 are 6,12,18 i.e., 3 in number.

∴ Numbers divisible by "2 or 3"=10+6 – 3=13

Favourable outcomes = 13

∴ Required probability = $\frac{13}{20}$

(iii) Prime numbers are 2,3,5,7,11,13,17,19 i.e., 8 in number

Favourable outcomes = 8

Required probability = $\frac{8}{20} = \frac{2}{5}$

****7. A box contains 3 blue marbles, 2 white marbles. If a marble is taken out at random from the box, what is the probability that it will be a white one? Blue one? Red one?**

Ans. Total no. of possible outcomes = 3+2+4 = 9

No. of favourable outcomes for white marbles = 2

Required probability = $\frac{2}{9}$

No. of favourable outcomes for blue marbles = 3

Required probability = $\frac{3}{9} = \frac{1}{3}$

No. of favourable outcomes for red marbles =4

Required probability = $\frac{4}{9}$

*****8. The integers from 1 to 30 inclusive are written on cards (one number on one card). These card one put in a box and well mixed. Joseph picked up one card. What is the probability that his card has**

(i) number 7 **(ii) an even number** **(iii) a prime number**

Ans. Total no. of possible outcomes = 30

(i) P (the no.7) = $\frac{1}{30}$

(ii) Even no. are 2,4,6,8,10,12,14,16,18,20,22,24,26,28,30

Favourable outcomes = 15

Required probability = $\frac{15}{30} = \frac{1}{2}$

(iii) Prime numbers from

1 to 30 are 2,3,5,7,11,13,17,19,23,29}

No. of favourable outcomes = 10

Required probability = $\frac{10}{30} = \frac{1}{3}$

SECTION D : LONG ANSWER : 5 MARKS

*****1. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting:**

(i) a king of red colour

(ii) a face card

(iii) a red face card

(iv) the jack of hearts

(v) a spade

Ans. Total number of favourable outcomes = 52

(i) There are two suits of red cards, i.e., diamond and heart. Each suit contains one king.

∴ Favourable outcomes = 1

Hence, P (a king of red colour) = $\frac{2}{52} = \frac{1}{26}$

(ii) There are 12 face cards in a pack.

∴ Favourable outcomes = 12

Hence, P (a face card) = $\frac{12}{52} = \frac{3}{13}$

(iii) There are two suits of red cards, i.e., diamond and heart. Each suit contains 3 face cards.

∴ Favourable outcomes = 2 x 3 = 6

Hence, P (a red face card) = $\frac{6}{52} = \frac{3}{26}$

(iv) There are only one jack of heart.

∴ Favourable outcome = 1

Hence, P (the jack of hearts) = $\frac{1}{52}$

(v) There are 13 cards of spade.

∴ Favourable outcomes = 13

Hence, P (a spade) = $\frac{13}{52} = \frac{1}{4}$

CASE STUDY BAESD QUESTION

Q. On a weekend Rani was playing cards with her family. The deck has 52 cards. If her brother drew one card.

1. Find the probability of getting a king of red colour.

Answer: 1/26

2. Find the probability of getting a face card.

Answer: 3/13

3. Find the probability of getting a queen of diamond?

Answer:1/52

V) UNSOLVED PRACTICE QUESTION PAPER

SECTIONS A: MCQ: 1MARKS

****1. The sum of the probabilities of all the elementary events of an experiment is**

(a) 0.5 (b) 1 (c) 2 (d) 1.5

****2. A card is selected at random from a well shuffled deck of 52 playing cards. The probability of its being a face card is**

(a) 3/13 (b) 4/13 (c) 6/13 (d) 9/13

*****3. If an event cannot occur, then its probability is**

(a) 1 (b) 3/4 (c) ½ (d) 0.

***4. An event is very unlikely to happen. Its probability is closest to**

(a) 0.0001 (b) 0.001 (c) 0.01 (d) 0.1

*****5. If P(A) denotes the probability of an event A, then**

(a) $P(A) < 0$ (b) $P(A) > 1$ (c) $0 \leq P(A) \leq 1$ (d) $-1 \leq P(A) \leq 1$

***6. The probability that a non leap year selected at random will contain 53 Sundays is**

(a) 1/7 (b) 2/7 (c) 3/7 (d) 5/7

***7. If the probability of an event is p, the probability of its complementary event will be**

(a) $p - 1$ (b) p (c) $1 - p$ (d) $1 - 1/p$

****8. A card is drawn from a deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is**

(a) 4 (b) 13 (c) 48 (d) 51

****9. The probability of getting a bad egg in a lot of 400 is 0.035. The number of bad eggs in the lot is**

(a) 7 (b) 14 (c) 21 (d) 28

*****10. Two dice are rolled simultaneously. What is the probability that 6 will come up at least once?**

(a) 1/6 (b) 7/36 (c) 11/36 (d) 13/36

MCQ ANSWERS: 1.b 2.a 3.d 4.a 5.c 6.a 7.c 8.d 9.b 10.c

SECTION B: SA I : 2 MARKS

****Q.1:** : 12 defective pens are accidentally mixed with 132 good ones. It is not possible to just look at a pen and tell whether or not it is defective. One pen is taken out at random from this lot. Determine the probability that the pen is taken out is a good one.

*Q.2: The probability of selecting a blue marble at random from a jar that contains only blue, black and green marbles is 1/5. The probability of selecting a black marble at random from the same jar is 1/4. If the jar contains 11 green marbles, find the total number of marbles in the jar?

****Q.3:** The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. What is the number of rotten apples in the heap?

***Q.4:** A bag contains 15 white and some black balls. If the probability of drawing a black ball from the bag is thrice that of drawing a white ball, find the number of black balls in the bag?

ANSWERS: Q1. 0.916 Q2. 20 Q3. 162 Q4. 45

SECTION C: SA II: 3 MARKS

**Q1.A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be:

(i) red? (ii) white? (iii) not green?

**Q2. Five cards – then ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random.

(i) What is the probability that the card is the queen?

(ii) If the queen is drawn and put aside, what is the probability that the second card picked up is (a) an ace? (b) a queen?

*Q3. . A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears

(i) a two-digit number

(ii) a perfect square number

(iii) a number divisible by 5.

*Q4. Which of the following experiments have equally likely outcomes? Explain.

(i) A driver attempts to start a car. The car starts or does not start.

(ii) A player attempts to shoot a basket ball, she/he shoots or misses the shot.

(iii) A baby is born. It is a boy or a girl.

**Q5. Find the probability that a number selected at random from the numbers 1,2,3,…35 is a

(i) prime number,

(ii) multiple of 7,

(iii) multiple of 3 or 5.

***Q6. Two coins are tossed simultaneously. What is the probability of getting

i) at least one head? Ii) at most one tail? Iii) a head and a tail?

***Q7. 15 Cards, numbered1,2,3,…..,15 are put in a box and mixed thoroughly. A card is drawn at random, find the probability that the card is

i) an even number?

ii) a number divisible by 2 or 3?

ANSWERS:

Q1.i)5/17, ii)8/17,iii) 13/17 Q2.i)1/5 ii)1/4, 0 Q3. i)81/90 ii)9/90 iii)18/90

Q4.only (iii) Q5.i)11/35, ii)5/35 , iii)16/35 Q6. i)3/4 ii)3/4 iii)1/2 Q7. i)7/15 ii)2/3

SECTION D : LONG ANSWER :5 MARKS

***Q1. A card is drawn at random from a well shuffled deck of playing cards. Find the probability that the card drawn is

(i) a red king

(ii) neither a king nor a queen

(iii) either a king or a queen

(iv) a face card

(v) cards which is neither king nor a red card.

Ans.

(i) 1/26 (ii) 11/13 (iii) 2/13 (iv) 3/13 (v) 6/13

CASE STUDY QUESTION

Q. Rahul and Ravi planned to play Business (board game) in which they were supposed to use two dice.

Q1. Ravi got first chance to roll the dice. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is 8?

Q2. Rahul got next chance. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is 13?

Q3. Now it was Ravi's turn. He rolled the dice. What is the probability that he got the sum of the two numbers appearing on the top face of the dice is less than or equal to 12?

Answer: Q1. 5/36 Q2.0 Q 3.1

****

CBSE SAMPLE PAPER

SAMPLE QUESTION PAPER

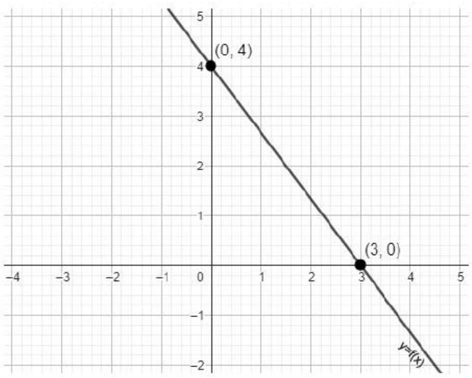
Class X Session 2023-24

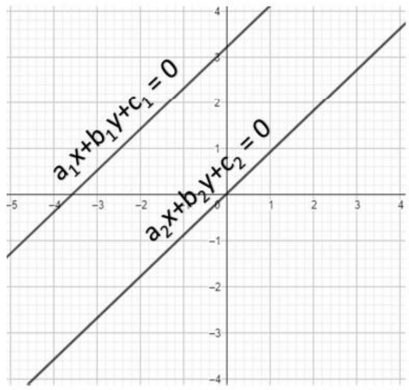
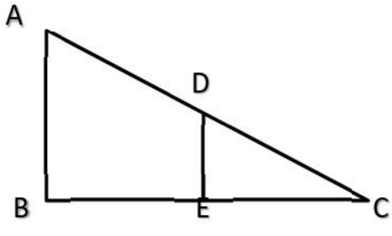
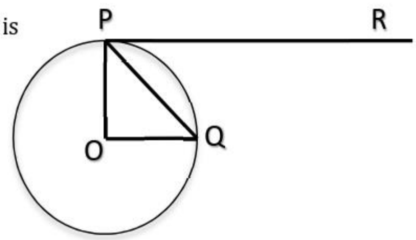
MATHEMATICS STANDARD (Code No.041)

TIME: 3 hours **MAX.MARKS: 80**

General Instructions:

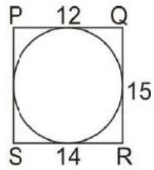
1. This Question Paper has 5 Sections A, B, C, D and E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

	SECTION A	
	Section A consists of 20 questions of 1 mark each.	
1.	If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$, where x, y are prime numbers, then the result obtained by dividing the product of the positive integers by the LCM (a, b) is (a) xy (b) xy^2 (c) x^3y^3 (d) x^2y^2	1
2.	The given linear polynomial y = f(x) has (a) 2 zeros (b) 1 zero and the zero is '3' (c) 1 zero and the zero is '4' (d) No zero 	1

3.	The lines representing the given pair of linear equations are non-intersecting. Which of the following statements is true?  (a) $\frac{a1}{a2}=\frac{b1}{b2}=\frac{c1}{c2}$ (b) $\frac{a1}{a2}=\frac{b1}{b2}\neq\frac{c1}{c2}$ (c) $\frac{a1}{a2}\neq\frac{b1}{b2}=\frac{c1}{c2}$ (d) $\frac{a1}{a2}\neq\frac{b1}{b2}\neq\frac{c1}{c2}$	1
4.	The nature of roots of the quadratic equation $9x^2 - 6x - 2 = 0$ is: (a) No real roots (b) 2 equal real roots (c) 2 distinct real roots (d) More than 2 real roots	1
5.	Two APs have the same common difference. The first term of one of these is –1 and that of the other is – 8. The difference between their 4th terms is (a) 1 (b) -7 (c) 7 (d) 9	1
6.	What is the ratio in which the line segment joining (2,-3) and (5, 6) is divided by x-axis? (a) 1:2 (b) 2:1 (c) 2:5 (d) 5:2	1
7.	A point (x,y) is at a distance of 5 units from the origin. How many such points lie in the third quadrant? (a) 0 (b) 1 (c) 2 (d) infinitely many	1
8.	In Δ ABC, DE || AB. If AB = a, DE = x, BE = b and EC = c. Then x expressed in terms of a, b and c is:  (a) $\frac{ac}{b}$ (b) $\frac{ac}{b+c}$ (c) $\frac{ab}{c}$ (d) $\frac{ab}{b+c}$	1
9.	If O is centre of a circle and Chord PQ makes an angle 50° with the tangent PR at the point of contact P, then the angle subtended by the chord at the centre is  (a) 130° (b) 100° (c) 50° (d) 30°	1

10. A quadrilateral PQRS is drawn to circumscribe a circle. (1)

If PQ = 12 cm, QR = 15 cm and RS = 14 cm, then find the length of SP is



(a) 15 cm (b) 14 cm

(b) (c) 12 cm (d) 11 cm

11. Given that $\sin\theta = \frac{a}{b}$, then $\cos\theta$ is. (1)

(a) $\frac{b}{\sqrt{b^2-a^2}}$ (b) $\frac{b}{a}$ (c) $\frac{\sqrt{b^2-a^2}}{b}$ (d) $\frac{a}{\sqrt{b^2-a^2}}$

12. (sec A + tan A) (1 – sin A) equals: (1)

(a) sec A (b) sin A (c) cosec A (d) cos A

13. If a pole 6 m high casts a shadow $2\sqrt{3}$m long on the ground, then the Sun's elevation is (1)

(a) 60° (b) 45° (c) 30° (d) 90°

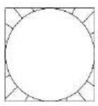
14. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is (1)

(a) 2 units (b) π units (c) 4 units (d) 7 units

15. It is proposed to build a new circular park equal in area to the sum of areas of two circular parks of diameters 16 m and 12 m in a locality. The radius of the new park is

(a) 10m (b) 15m (c) 20m (d) 24m

16. There is a square board of side '2a' units circumscribing a red circle. Jayadev is asked to keep a dot on the above said board. The probability that he keeps the dot on the shaded region is. (1)



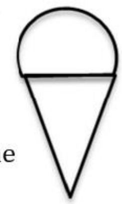
(a) $\frac{\pi}{4}$ (b) $\frac{4-\pi}{4}$ (c) $\frac{\pi-4}{4}$ (d) $\frac{4}{\pi}$

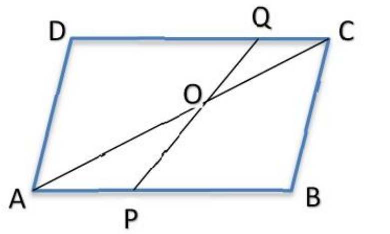
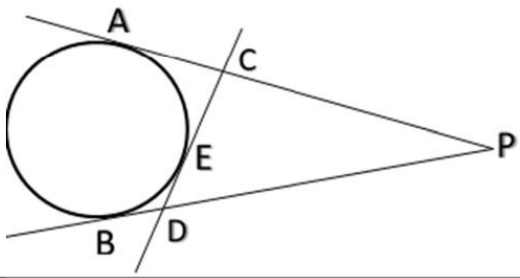
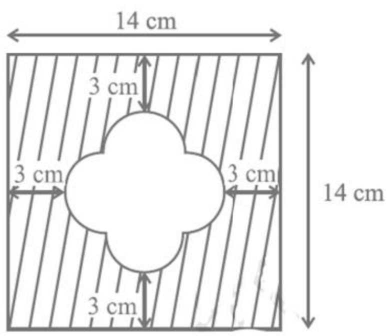
17. 2 cards of hearts and 4 cards of spades are missing from a pack of 52 cards. A card is drawn at random from the remaining pack. What is the probability of getting a black card? (1)

(a) $\frac{22}{52}$ (b) $\frac{22}{46}$ (c) $\frac{24}{52}$ (d) $\frac{24}{46}$

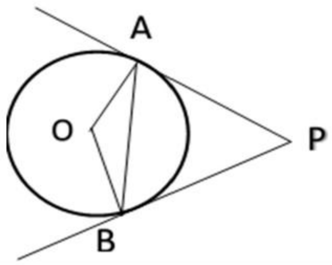
18. The upper limit of the modal class of the given distribution is: (1)

Height [in cm]	Below 140	Below 145	Below 150	Below 155	Below 160	Below 165
Number of girls	4	11	29	40	46	51

	(a) 165 (b) 160 (c) 155 (d) 150	
19.	DIRECTION: In the question number 19 and 20, a statement of assertion (A) is followed by a statement of Reason (R). Choose the correct option Statement A (Assertion): Total Surface area of the top is the sum of the curved surface area of the hemisphere and the curved surface area of the cone. Statement R(Reason) : Top is obtained by joining the plane surfaces of the hemisphere and cone together.  (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A) (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) (c) Assertion (A) is true but reason (R) is false. (d) Assertion (A) is false but reason (R) is true.	1
20.	Statement A (Assertion): -5, $\frac{-5}{2}$, 0, $\frac{5}{2}$, …. is in Arithmetic Progression. Statement R (Reason) : The terms of an Arithmetic Progression cannot have both positive and negative rational numbers. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A) (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A) (c) Assertion (A) is true but reason (R) is false. (d) Assertion (A) is false but reason (R) is true.	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21.	Prove that $\sqrt{2}$ is an irrational number.	2

22.	ABCD is a parallelogram. Point P divides AB in the ratio 2:3 and point Q divides DC in the ratio 4:1. Prove that OC is half of OA. 	2
23.	From an external point P, two tangents, PA and PB are drawn to a circle with centre O. At a point E on the circle, a tangent is drawn to intersect PA and PB at C and D, respectively. If PA = 10 cm, find the perimeter of ΔPCD. 	2
24.	If $\tan(A + B) = \sqrt{3}$ and $\tan(A - B) = \frac{1}{\sqrt{3}}$; $0° < A + B < 90°$; $A > B$, find A and B.	2
	[or]	
	Find the value of x if $2\,\text{cosec}^2 30 + x \sin^2 60 - \frac{3}{4}\tan^2 30 = 10$	
25.	With vertices A, B and C of ΔABC as centres, arcs are drawn with radii 14 cm and the three portions of the triangle so obtained are removed. Find the total area removed from the triangle.	2
	[or]	
	Find the area of the unshaded region shown in the given figure. 	
	SECTION C	
	Section C consists of 6 questions of 3 marks each	
26.	National Art convention got registrations from students from all parts of the country, of which 60 are interested in music, 84 are interested in dance and 108 students are interested	3

Page **5** of **10**

	in handicrafts. For optimum cultural exchange, organisers wish to keep them in minimum number of groups such that each group consists of students interested in the same artform and the number of students in each group is the same. Find the number of students in each group. Find the number of groups in each art form. How many rooms are required if each group will be allotted a room?	
27.	If α, β are zeroes of quadratic polynomial $5x^2 + 5x + 1$, find the value of 1. $\alpha^2 + \beta^2$ 2. $\alpha^{-1} + \beta^{-1}$	3
28.	The sum of a two digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?	3
	[or]	
	Solve : - $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$; $\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$, x, y>o	
29.	PA and PB are tangents drawn to a circle of centre O from an external point P. Chord AB makes an angle of 30° with the radius at the point of contact. If length of the chord is 6 cm, find the length of the tangent PA and the length of the radius OA.  	3
	[or]	
	Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠ PTQ = 2 ∠ OPQ.	
30.	If 1 + sin²θ = 3sinθ cosθ , then prove that tanθ = 1 or $\frac{1}{2}$	3
31.	The length of 40 leaves of a plant are measured correct to nearest millimetre, and the data obtained is represented in the following table.	3

Length [in mm]	Number of leaves
118 – 126	3
127 – 135	5
136 – 144	9

145 – 153	12
154 – 162	5
163 – 171	4
172 – 180	2

Find the mean length of the leaves.

SECTION D

Section D consists of 4 questions of 5 marks each

32. A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of stream. 5

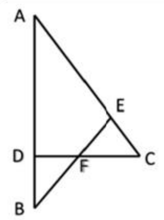
[or]

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

33. (a) State and prove Basic Proportionality theorem. 5

(b) In the given figure $\angle CEF = \angle CFE$. F is the midpoint of DC.

Prove that $\frac{AB}{BD} = \frac{AE}{FD}$



34. Water is flowing at the rate of 15 km/h through a pipe of diameter 14 cm into a cuboidal pond which is 50 m long and 44 m wide. In what time will the level of water in pond rise by 21 cm? 5

What should be the speed of water if the rise in water level is to be attained in 1 hour?

[or]

A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 3 m and 14 m respectively, and the total height of the tent is 13.5 m, find the area of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of ₹ 500 per m^2.

35.	The median of the following data is 50. Find the values of 'p' and 'q', if the sum of all frequencies is 90. Also find the mode of the data.	5

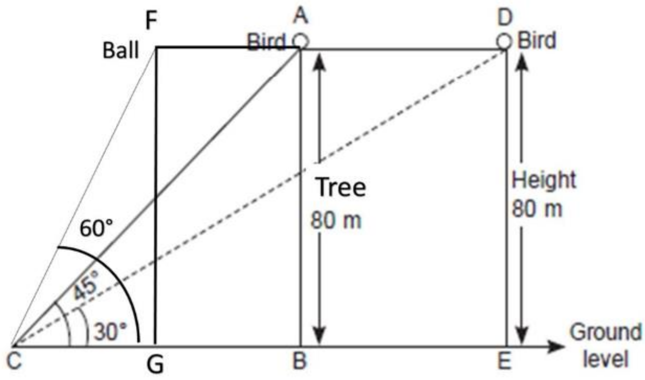
Marks obtained	Number of students
20 – 30	p
30 – 40	15
40 – 50	25
50 – 60	20
60 – 70	q
70 – 80	8
80 - 90	10

SECTION E

36. Manpreet Kaur is the national record holder for women in the shot-put discipline. Her throw of 18.86m at the Asian Grand Prix in 2017 is the maximum distance for an Indian female athlete.
Keeping her as a role model, Sanjitha is determined to earn gold in Olympics one day.
Initially her throw reached 7.56m only. Being an athlete in school, she regularly practiced both in the mornings and in the evenings and was able to improve the distance by 9cm every week.
During the special camp for 15 days, she started with 40 throws and every day kept increasing the number of throws by 12 to achieve this remarkable progress.

	(i) How many throws Sanjitha practiced on 11th day of the camp?	1
	(ii) What would be Sanjitha's throw distance at the end of 6 weeks? (or) When will she be able to achieve a throw of 11.16 m?	2
	(iii) How many throws did she do during the entire camp of 15 days ?	1
37.	Tharunya was thrilled to know that the football tournament is fixed with a monthly timeframe from 20th July to 20th August 2023 and for the first time in the FIFA Women's World Cup's history, two nations host in 10 venues. Her father felt that the game can be better understood if the position of players is represented as points on a coordinate plane.	

			
	(i)	At an instance, the midfielders and forward formed a parallelogram. Find the position of the central midfielder (D) if the position of other players who formed the parallelogram are :- A(1,2), B(4,3) and C(6,6)	1
	(ii)	Check if the Goal keeper G(-3,5), Sweeper H(3,1) and Wing-back K(0,3) fall on a same straight line. [or] Check if the Full-back J(5,-3) and centre-back I(-4,6) are equidistant from forward C(0,1) and if C is the mid-point of IJ.	2
	(iii)	If Defensive midfielder A(1,4), Attacking midfielder B(2,-3) and Striker E(a,b) lie on the same straight line and B is equidistant from A and E, find the position of E.	1
38.		One evening, Kaushik was in a park. Children were playing cricket. Birds were singing on a nearby tree of height 80m. He observed a bird on the tree at an angle of elevation of 45°. When a sixer was hit, a ball flew through the tree frightening the bird to fly away. In 2 seconds, he observed the bird flying at the same height at an angle of elevation of 30° and the ball flying towards him at the same height at an angle of elevation of 60°.	

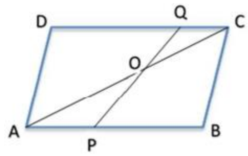
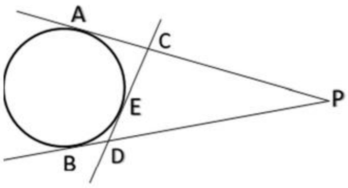
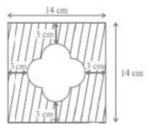
		
	(i) At what distance from the foot of the tree was he observing the bird sitting on the tree?	1
	(ii) How far did the bird fly in the mentioned time? (or) After hitting the tree, how far did the ball travel in the sky when Kaushik saw the ball?	2
	(iii) What is the speed of the bird in m/min if it had flown $20(\sqrt{3}+1)$ m?	1

Marking Scheme
Class X Session 2023-24
MATHEMATICS STANDARD (Code No.041)

TIME: 3 hours **MAX.MARKS: 80**

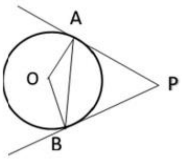
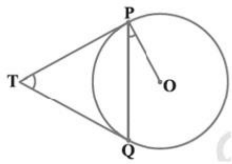
	SECTION A	
	Section A consists of 20 questions of 1 mark each.	
1.	(b) xy^2	1
2.	(b) 1 zero and the zero is '3'	1
3.	(b) $\frac{a1}{a2} = \frac{b1}{b2} \neq \frac{c1}{c2}$	1
4.	(c) 2 distinct real roots	1
5.	(c) 7	1
6.	(a) 1:2	1
7.	(d) infinitely many	1
8.	(b) $\frac{ac}{b+c}$	1
9.	(b) 100°	1
10.	(d) 11 cm	1
11.	(c) $\frac{\sqrt{b^2-a^2}}{b}$	1
12.	(d) cos A	1
13.	(a) 60°	1
14.	(a) 2 units	1
15.	(a) 10m	1
16.	(b) $\frac{4-\pi}{4}$	1
17.	(b) $\frac{22}{46}$	1
18.	(d) 150	1
19.	(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)	1
20.	(c) Assertion (A) is true but reason (R) is false.	1
	SECTION B	
	Section B consists of 5 questions of 2 marks each.	
21.	Let us assume, to the contrary, that $\sqrt{2}$ is rational. So, we can find integers a and b such that $\sqrt{2} = \frac{a}{b}$ where a and b are coprime.	½
	So, b $\sqrt{2}$ = a. Squaring both sides, we get $2b^2 = a^2$. Therefore, 2 divides a^2 and so 2 divides a.	½
	So, we can write a = 2c for some integer c. Substituting for a, we get $2b^2 = 4c^2$, that is, $b^2 = 2c^2$. This means that 2 divides b^2, and so 2 divides b	½
	Therefore, a and b have at least 2 as a common factor. But this contradicts the fact that a and b have no common factors other than 1. This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational. So, we conclude that $\sqrt{2}$ is irrational.	½

1

22.	ABCD is a parallelogram. AB = DC = a Point P divides AB in the ratio 2:3 AP = $\frac{2}{5}$ a , BP = $\frac{3}{5}$ a point Q divides DC in the ratio 4:1. DQ = $\frac{4}{5}$ a , CQ = $\frac{1}{5}$ a Δ APO ~ Δ CQO [AA similarity] $\frac{AP}{CQ} = \frac{PO}{QO} = \frac{AO}{CO}$ $\frac{AO}{CO} = \frac{\frac{2}{5}a}{\frac{1}{5}a} = \frac{2}{1} \Rightarrow$ OC = ½ OA  	½ ½ ½ ½
23.	PA = PB; CA = CE; DE = DB [Tangents to a circle] Perimeter of ΔPCD = PC + CD + PD = PC + CE + ED + PD = PC + CA + BD + PD = PA + PB Perimeter of ΔPCD = PA + PA = 2PA = 2(10) = 20 cm  	½ 1 ½
24.	$\because \tan(A+B) = \sqrt{3} \quad \therefore A + B = 60^0$ ...(1) $\because \tan(A-B) = \frac{1}{\sqrt{3}} \quad \therefore A - B = 30^0$ ...(2) Adding (1) & (2), we get 2A=$90^0 \Longrightarrow A = 45^0$ Also (1) –(2), we get $2B = 30^0 \Longrightarrow B = 45^0$	½ ½ ½ ½
	[or]	
	2 cosec²30 + x sin²60 – $\frac{3}{4}$ tan²30 = 10 $\Rightarrow \quad 2(2)^2 + x\left(\frac{\sqrt{3}}{2}\right)^2 - \frac{3}{4}\left(\frac{1}{\sqrt{3}}\right)^2 = 10$ $\Rightarrow \quad 2(4) + x\left(\frac{3}{4}\right) - \frac{3}{4}\left(\frac{1}{3}\right) = 10$ $\Rightarrow \quad 8 + x\left(\frac{3}{4}\right) - \frac{1}{4} = 10$ $\Rightarrow \quad 32 + x(3) - 1 = 40$ $\Rightarrow \quad 3x = 9 \quad \Rightarrow \quad x = 3$	1 ½ ½
25.	Total area removed = $\frac{\angle A}{360}\pi r^2 + \frac{\angle B}{360}\pi r^2 + \frac{\angle C}{360}\pi r^2$ $= \frac{\angle A + \angle B + \angle C}{360}\pi r^2$ $= \frac{180}{360}\pi r^2$ $= \frac{180}{360} X \frac{22}{7} X (14)^2$ ½ = 308 cm²	½ ½ ½
	[or]	
	The side of a square = Diameter of the semi-circle = a Area of the unshaded region = Area of a square of side 'a' + 4(Area of a semi-circle of diameter 'a') The horizontal/vertical extent of the white region = 14-3-3 = 8 cm Radius of the semi-circle + side of a square + Radius of the semi-circle = 8 cm  	½ ½

2

	2 (radius of the semi-circle) + side of a square = 8 cm $2a = 8$ cm $\Rightarrow a = 4$ cm Area of the unshaded region = Area of a square of side 4 cm + 4 (Area of a semi-circle of diameter 4 cm) $= (4)^2 + 4 \times \frac{1}{2}\pi(2)^2 = (16 + 8\pi)$ cm^2	½ ½
	SECTION C	
	Section C consists of 6 questions of 3 marks each	
26.	Number of students in each group subject to the given condition = HCF (60,84,108) HCF (60,84,108) = 12 Number of groups in Music = $\frac{60}{12} = 5$ Number of groups in Dance = $\frac{84}{12} = 7$ Number of groups in Handicrafts = $\frac{108}{12} = 9$ Total number of rooms required = 21	½ ½ ½ ½ ½ ½
27.	$P(x) = 5x^2 + 5x + 1$ $\alpha+\beta = \frac{-b}{a} = \frac{-5}{5} = -1$ $\alpha\beta = \frac{c}{a} = \frac{1}{5}$ $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ $= (-1)^2 - 2\left(\frac{1}{5}\right)$ $= 1 - \frac{2}{5} = \frac{3}{5}$ $\alpha^{-1} + \beta^{-1} = \frac{1}{\alpha} + \frac{1}{\beta}$ $= \frac{(\alpha+\beta)}{\alpha\beta} = \frac{(-1)}{\frac{1}{5}} = -5$	½ ½ ½ ½ ½ ½
28.	Let the ten's and the unit's digits in the first number be x and y, respectively. So, the original number = 10x + y When the digits are reversed, x becomes the unit's digit and y becomes the ten's Digit. So the obtain by reversing the digits= 10y + x According to the given condition. (10x + y) + (10y + x) = 66 i.e., 11(x + y) = 66 i.e., x + y = 6 ---- (1) We are also given that the digits differ by 2, therefore, either x – y = 2 ---- (2) or y – x = 2 ---- (3) If x – y = 2, then solving (1) and (2) by elimination, we get x = 4 and y = 2. In this case, we get the number 42. If y – x = 2, then solving (1) and (3) by elimination, we get x = 2 and y = 4. In this case, we get the number 24. Thus, there are two such numbers 42 and 24.	½ ½ ½ ½ ½ ½
	[or]	
	Let $\frac{1}{\sqrt{x}}$ be 'm' and $\frac{1}{\sqrt{y}}$ be 'n', Then the given equations become 2m + 3n = 2 4m - 9n = -1	½ ½

	(2m + 3n = 2) X- 2 $\Rightarrow -4m - 6n = -4$ ...(1) 4m - 9n = -1 $4m - 9n = -1$ ...(2) Adding (1) and (2) We get $-15n = -5 \Rightarrow n = \frac{1}{3}$ Substituting n = $\frac{1}{3}$ in 2m + 3n = 2, we get 2m + 1 = 2 2m = 1 m = $\frac{1}{2}$ m = $\frac{1}{2}$ $\Rightarrow \sqrt{x} = 2 \Rightarrow$ x = 4 and n = $\frac{1}{3}$ $\Rightarrow \sqrt{y} = 3 \Rightarrow$ y = 9	½ ½ 1
29.	 $\angle$OAB = 30° $\angle$OAP = 90° [Angle between the tangent and the radius at the point of contact] $\angle$PAB = 90° - 30° =60° AP = BP [Tangents to a circle from an external point] $\angle$PAB = $\angle$PBA [Angles opposite to equal sides of a triangle] In ΔABP, $\angle$PAB + $\angle$PBA + $\angle$APB = 180° [Angle Sum Property] 60° + 60° + $\angle$APB = 180° $\angle$APB = 60° ∴ ΔABP is an equilateral triangle, where AP = BP = AB. PA = 6 cm In Right ΔOAP, $\angle$OPA = 30° tan 30° = $\frac{OA}{PA}$ $\frac{1}{\sqrt{3}} = \frac{OA}{6}$ OA = $\frac{6}{\sqrt{3}}$ = $2\sqrt{3}cm$	½ ½ ½ ½ ½ ½
	[or]	
	 Let $\angle$ TPQ $= \theta$ $\angle$ TPO = 90° [Angle between the tangent and the radius at the point of contact] $\angle$ OPQ = 90° - θ TP = TQ [Tangents to a circle from an external point] $\angle$ TPQ = $\angle$ TQP = θ [Angles opposite to equal sides of a triangle] In ΔPQT, $\angle$PQT + $\angle$QPT + $\angle$PTQ = 180° [Angle Sum Property] $\theta + \theta$ + $\angle$PTQ = 180° $\angle$PTQ = 180° - 2 θ $\angle$PTQ = 2 (90° - θ) $\angle$PTQ = 2 $\angle$ OPQ [using (1)]	½ ½ ½ ½ ½ ½
30.	Given, 1 + sin²θ = 3 sin θ cos θ Dividing both sides by cos²θ, $\frac{1}{\cos^2\theta}$ + tan²θ = 3 tan θ sec²θ + tan²θ = 3 tan θ 1 + tan²θ + tan²θ = 3 tan θ 1 + 2 tan²θ = 3 tan θ 2 tan²θ - 3 tan θ +1 = 0 If tan θ = x, then the equation becomes 2x² -3x + 1 = 0	½ ½ ½ ½

4

$\Rightarrow (x-1)(2x-1) = 0$ x = 1 or $\frac{1}{2}$

tan θ = 1 or $\frac{1}{2}$ 1

31.

Length [in mm]	Number of leaves (f)	CI	Mid x	d	fd
118 – 126	3	117.5- 126.5	122	-27	-81
127 – 135	5	126.5– 135.5	131	-18	-90
136 – 144	9	135.5– 144.5	140	-9	-81
145 – 153	12	144.5 – 153.5	a = 149	0	0
154 – 162	5	153.5 – 162.5	158	9	45
163 – 171	4	162.5 – 171.5	167	18	72
172 – 180	2	171.5 – 180.5	176	27	54

2

½

½

Mean = a + $\frac{\sum fd}{\sum f}$ = 149 + $\frac{-8}{40}$

= 149 – 2.025 = 146.975

Average length of the leaves = 146.975

SECTION D

Section D consists of 4 questions of 5 marks each

32. Let the speed of the stream be x km/h.

The speed of the boat upstream = (18 – x) km/h and

the speed of the boat downstream = (18 + x) km/h. 1

The time taken to go upstream = $\frac{distance}{speed} = \frac{24}{18-x}$ hours

the time taken to go downstream = $\frac{distance}{spe} = \frac{24}{18+x}$ hours 1

According to the question,

$$\frac{24}{18-x} - \frac{24}{18+x} = 1$$ 1

24(18 + x) – 24(18 – x) = (18 – x) (18 + x)

x^2 + 48x – 324 = 0

x = 6 or – 54 1

Since x is the speed of the stream, it cannot be negative.

Therefore, x = 6 gives the speed of the stream = 6 km/h. 1

[or]

Let the time taken by the smaller pipe to fill the tank = x hr.

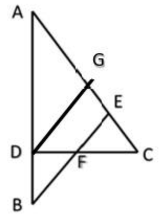
Time taken by the larger pipe = (x – 10) hr ½

Part of the tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of the tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$ 1

The tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together. ½

Part of the tank filled by both the pipes in 1 hour = $\frac{8}{75}$ ½

	Therefore, $\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$ $8x^2 - 230x + 750 = 0$ $x = 25, \frac{30}{8}$ Time taken by the smaller pipe cannot be 30/8 = 3.75 hours, as the time taken by the larger pipe will become negative, which is logically not possible. Therefore, the time taken individually by the smaller pipe is 25 hours and the larger pipe will be 25 – 10 =15 hours.	½ 1 ½ ½
33.	(a) Statement – ½ Given and To Prove – ½ Figure and Construction ½ Proof – 1 ½  [b] Draw DG ‖ BE In Δ ABE, $\frac{AB}{BD} = \frac{AE}{GE}$ [BPT] CF = FD [F is the midpoint of DC] ---(i) In Δ CDG, $\frac{DF}{CF} = \frac{GE}{CE} = 1$ [Mid point theorem] GE = CE ---(ii) ∠CEF = ∠CFE [Given] CF = CE [Sides opposite to equal angles] ---(iii) From (ii) & (iii) CF = GE ---(iv) From (i) & (iv) GE = FD $\therefore \frac{AB}{BD} = \frac{AE}{GE} \Rightarrow \frac{AB}{BD} = \frac{AE}{FD}$	3 ½ ½ ½ ½
34.	Length of the pond, l= 50m, width of the pond, b = 44m Water level is to rise by, h = 21 cm = $\frac{21}{100}$ m Volume of water in the pond = lbh = 50 x 44 x $\frac{21}{100}$ m^3 =462 m^3 Diameter of the pipe = 14 cm Radius of the pipe, r = 7cm = $\frac{7}{100}$ m Area of cross-section of pipe = πr^2 $= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} = \frac{154}{10000}$ m^2 Rate at which the water is flowing through the pipe, h = 15km/h = 15000 m/h Volume of water flowing in 1 hour = Area of cross-section of pipe x height of water coming out of pipe $= \left(\frac{154}{10000} \times 15000\right) m^3$ Time required to fill the pond $= \frac{Volume\ of\ the\ pond}{Volume\ of\ water\ flowing\ in\ 1\ hour}$ $= \frac{462 \times 10000}{154 \times 15000} = 2$ hours Speed of water if the rise in water level is to be attained in 1 hour = 30km/h	1 1 ½ ½ 1 1
	[or]	

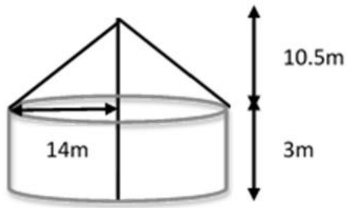
6

Radius of the cylindrical tent (r) = 14 m

Total height of the tent = 13.5 m

Height of the cylinder = 3 m

Height of the Conical part = 10.5 m ½



Slant height of the cone (l) = $\sqrt{h^2 + r^2}$

$= \sqrt{(10.5)^2 + (14)^2}$

$= \sqrt{110.25 + 196}$

$= \sqrt{306.25} = 17.5$ m 1

Curved surface area of cylindrical portion

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 14 \times 3$ 1

$= 264 \text{ m}^2$

Curved surface area of conical portion

$= \pi rl$

$= \frac{22}{7} \times 14 \times 17.5$ 1

$= 770 \text{ m}^2$ ½

Total curved surface area = $264 \text{ m}^2 + 770 \text{ m}^2 = 1034 \text{ m}^2$

Provision for stitching and wastage = 26 m^2 ½

Area of canvas to be purchased = 1060 m^2

Cost of canvas = Rate × Surface area ½

= 500 x 1060 = ₹ 5,30,000/-

35.

Marks obtained	Number of students	Cumulative frequency
20 – 30	p	p
30 – 40	15	p + 15
40 – 50	25	p + 40
50 – 60	20	p + 60
60 – 70	q	p + q + 60
70 – 80	8	p + q + 68
80 - 90	10	p + q + 78
	90	

1

½

½

p + q + 78 = 90

p + q = 12

Median $= (l) + \frac{\frac{n}{2} - c}{f} \cdot h$

$50 = 50 + \frac{45-(p+40)}{20} \cdot 10$ ½

$\frac{45-(p+40)}{20} \cdot 10 = 0$ ½

$45 - (p + 40) = 0$

P = 5 ½

5 + q = 12 ½

q = 7

Mode $= l + \frac{f1-f0}{2f1-f0-f2} \cdot h$ 1

7

	$= 40 + \frac{25-15}{2(25)-15-20} \cdot 10$ $= 40 + \frac{100}{15} = 40 + 6.67 = 46.67$	
	SECTION E	
36.	(i) Number of throws during camp. a = 40; d = 12 $t_{11} = a + 10d$ $= 40 + 10 \times 12$ $= 160\ throws$	1
	(ii) a = 7.56 m; d = 9cm = 0.09 m n = 6 weeks t_n = a + (n-1) d = 7.56 + 6(0.09) = 7.56 + 0.54 Sanjitha's throw distance at the end of 6 weeks = 8.1 m (or) a = 7.56 m; d = 9cm = 0.09 m t_n =11.16 m t_n = a + (n-1) d 11.16 = 7.56 + (n-1) (0.09) 3.6 = (n-1) (0.09) n-1 = $\frac{3.6}{0.09}$ = 40 n = 41 Sanjitha's will be able to throw 11.16 m in 41 weeks.	½ ½ ½ ½ ½ ½ ½ ½
	(iii) a = 40; d = 12; n = 15 $S_n = \frac{n}{2}$ [2a + (n-1) d] $S_n = \frac{15}{2}$ [2(40) + (15-1) (12)] $= \frac{15}{2}$ [80 + 168] $= \frac{15}{2}$ [248] =1860 throws	½ ½
37.	(i) Let D be (a,b), then Mid point of AC = Midpoint of BD $\left(\frac{1+6}{2}, \frac{2+6}{2}\right) = \left(\frac{4+a}{2}, \frac{3+b}{2}\right)$ 4 + a = 7 3+ b = 8 a = 3 b = 5 Central midfielder is at (3,5)	½ ½

8

	(ii) GH = $\sqrt{(-3-3)^2+(5-1)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$ GK = $\sqrt{(0+3)^2+(3-5)^2} = \sqrt{9+4} = \sqrt{13}$ HK = $\sqrt{(3-0)^2+(1-3)^2} = \sqrt{9+4} = \sqrt{13}$ GK +HK = GH ⇒G,H & K lie on a same straight line [or] CJ = $\sqrt{(0-5)^2+(1+3)^2} = \sqrt{25+16} = \sqrt{41}$ CI = $\sqrt{(0+4)^2+(1-6)^2} = \sqrt{16+25} = \sqrt{41}$ Full-back J(5,-3) and centre-back I(-4,6) are equidistant from forward C(0,1) Mid-point of IJ = $\left(\frac{5-4}{2}, \frac{-3+6}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$ C is NOT the mid-point of IJ	½ ½ ½ ½ ½ ½ ½ ½
	(iii) A,B and E lie on the same straight line and B is equidistant from A and E ⇒ B is the mid-point of AE $\left(\frac{1+a}{2}, \frac{4+b}{2}\right) = (2, -3)$ $1 + a = 4$; a = 3. 4+b = -6; b = -10 E is (3,-10)	½ ½
38.	(i) tan 45° = $\frac{80}{CB}$ ⇒ CB = 80m	1
	(ii) tan 30° = $\frac{80}{CE}$ ⇒ $\frac{1}{\sqrt{3}} = \frac{80}{CE}$ ⇒ CE = $80\sqrt{3}$ Distance the bird flew = AD = BE = CE-CB = $80\sqrt{3} - 80 = 80(\sqrt{3}-1)$ m (or) tan 60° = $\frac{80}{CG}$ ⇒ $\sqrt{3} = \frac{80}{CG}$ ⇒ CG = $\frac{80}{\sqrt{3}}$ Distance the ball travelled after hitting the tree =FA=GB = CB -CG GB = 80 - $\frac{80}{\sqrt{3}} = 80\left(1 - \frac{1}{\sqrt{3}}\right)$ m	½ ½ ½ ½ ½ ½ ½ ½
	(iii) Speed of the bird = $\frac{Distance}{Time\ taken} = \frac{20(\sqrt{3}+1)}{2}$ m/sec $= \frac{20(\sqrt{3}+1)}{2}$ X 60 m/min = $600(\sqrt{3}+1)$ m/min	½ ½

9

Sample Question Paper (2023-24)
Class – X
Basic Mathematics (241)

Time Allowed: 3 Hrs **Maximum Marks: 80**

General Instructions:

1. **This Question Paper has 5 Sections A, B, C, D, and E.**
2. **Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.**
3. **Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.**
4. **Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.**
5. **Section D has 4 Long Answer (LA) type questions carrying 5 marks each.**
6. **Section E has 3 sourced based/Case Based/passage based/integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.**
7. **All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.**
8. **Draw neat figures wherever required. Take π =22/7 wherever required if not stated.**

SECTION A

1. If two positive integers a and b are written as $a = x^3y^2$ and $b = xy^3$; where x, y are prime numbers, then HCF (a,b) is:

   a) xy b) xy^2 c) x^3y^3 d) x^2y^2

2. The LCM of smallest two digit composite number and smallest composite number is:

   a) 12 b) 4 c) 20 d) 44

3. If x = 3 is one of the roots of the quadratic equation $x^2 – 2kx – 6 = 0$, then the value of k is

   a) $-\frac{1}{2}$ b) $\frac{1}{2}$ c) 3 d) 2

4. The pair of equations y = 0 and y = -7 has:

   a) one solution b) two solutions c) infinitely many solutions d) no solution

5. Value(s) of k for which the quadratic equation $2x^2 – kx + k = 0$ has equal roots is :

   a) 0 only b) 4 c) 8 only d) 0,8

6. The distance of the point(3, 5) from x-axis is k units, then k equals:

   a) 3 b) 4 c) 5 d) 8

7. If in Δ ABC and Δ PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$ then:

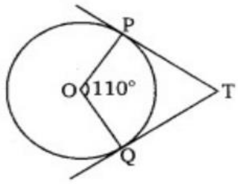
   a) ΔPQR ∼ΔCAB b) ΔPQR ∼ΔABC c) ΔCBA ∼ΔPQR d) ΔBCA ∼ΔPQR

1

8. Which of the following is NOT a similarity criterion of traingles?

a) AA b) SAS c) AAA d) RHS

9. In figure, if TP and TQ are the two tangents to a circle with centre O so that ∠POQ = 110°, then ∠PTQ is equal to



(a) 60° (b) 70° (c) 80° (d) 90°

10. If cos A = $\frac{4}{5}$ then tan A is:

a) $\frac{3}{5}$ b) $\frac{3}{4}$ c) $\frac{4}{3}$ d) $\frac{1}{8}$

11. If the height of the tower is equal to the length of its shadow, then the angle of elevation of the sun is _____

a) 30° b) 45° c) 60° d) 90°

12. $(1 - \cos^2 A)$ is equal to

a) $\sin^2 A$ b) $\tan^2 A$ c) $1 - \sin^2 A$ d) $\sec^2 A$

13. The radius of a circle is same as the side of a square. Their perimeters are in the ratio

a) 1 : 1 b) 2 : π c) π : 2 d) $\sqrt{\pi}$: 2

14. The area of the circle is 154cm^2. The radius of the circle is

a) 7cm b) 14cm c) 3.5cm d) 17.5cm

15. When a dice is thrown once, the probability of getting an even number less than 4 is

a) 1/4 b) 0 c) 1/2 d) 1/6

16. For the following distribution:

Class	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

The lower limit of modal class is:

a) 15 b) 20 c) 10 d) 5

17. A rectangular sheet of paper 40cm x 22cm, is rolled to form a hollow cylinder of height 40cm. The radius of the cylinder(in cm) is :

a) 3.5 b) 7 c) $\frac{80}{7}$ d) 5

18. Consider the following frequency distribution:

Class	0-6	6-12	12-18	18-24	24-30
Frequency	12	10	15	8	11

2

The median class is:

a) 6-12 b) 12-18 c) 18-24 d) 24-30

19. Assertion (A): The point (0, 4) lies on y-axis.

Reason(R): The x-coordinate of a point on y-axis is zero

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertions (A) is true but reason (R) is false.

(d) Assertions (A) is false but reason (R) is true.

20. Assertion (A): The HCF of two numbers is 5 and their product is 150. Then their LCM is 40.

Reason(R): For any two positive integers a and b, HCF (a, b) x LCM (a, b) = a x b.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertions (A) is true but reason (R) is false.

(d) Assertions (A) is false but reason (R) is true.

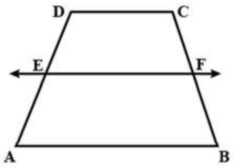
SECTION B

21. Find whether the following pair of linear equations is consistent or inconsistent:

$3x + 2y = 8$

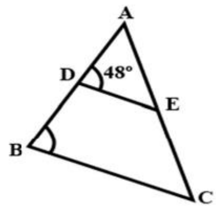
$6x - 4y = 9$

22. In the given figure, if ABCD is a trapezium in which AB ∥ CD ∥ EF, then prove that $\frac{AE}{ED} = \frac{BF}{FC}$.



3

OR



In figure, if AD = 6cm, DB = 9cm, AE = 8cm and EC = 12cm and ∠ADE = 48˚. Find ∠ABC.

23. The length of a tangent from a point A at distance 5cm from the centre of the circle is 4cm. Find the radius of the circle.

24. Evaluate: $\sin^2 60^\circ + 2\tan 45^\circ - \cos^2 30^\circ$.

25. Find the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40cm and 9cm.

OR

A chord of a circle of radius 10cm subtends a right angle at the centre. Find the area of minor segment. (Use π = 3.14)

SECTION C

26. Prove that $\sqrt{3}$ is an irrational number.

27. Find the zeroes of the quadratic polynomial $4s^2 - 4s + 1$ and verify the relationship between the zeroes and the coefficients.

28. The coach of a cricket team buys 4 bats and 1 ball for Rs. 2050. Later, she buys 3 bats and 2 balls for ₹1600. Find the cost of each bat and each ball.

OR

A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid ₹27 for a book kept for seven days, while Susy paid ₹21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

29. A circle touches all the four sides of quadrilateral ABCD. Prove that AB + CD = AD + BC.

30. Prove that

$$(\text{cosec}\,\theta - \cot\theta)^2 = \frac{1-\cos}{1+\cos\theta}$$

4

OR

Prove that sec A (1 – sin A) (sec A + tan A) = 1.

31. A bag contains 6 red, 4 black and some white balls.

(i) Find the number of white balls in the bag if the probability of drawing a white ball is $\frac{1}{3}$.

(ii) How many red balls should be removed from the bag for the probability of drawing a white ball to be $\frac{1}{2}$?

SECTION D

32. A train travels 360km at a uniform speed. If the speed had been 5km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

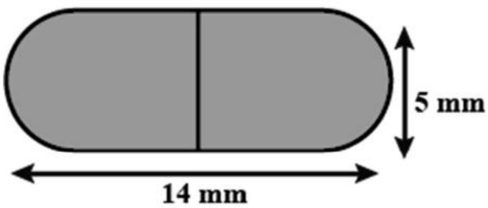
OR

A motor boat whose speed is 18km/h in still water takes 1 hour more to go 24km upstream than to return downstream to the same spot. Find the speed of the stream.

33. Prove that If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

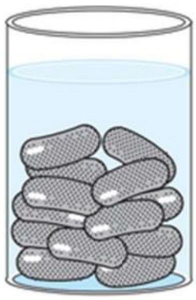
In ΔPQR, S and T are points on PQ and PR respectively. $\frac{PS}{SQ} = \frac{PT}{TR}$ and ∠PST = ∠PRQ. Prove that PQR is an isosceles triangle.

34. A medicine capsule is in the shape of a cylinder with two hemispheres stuck at each of its ends. The length of the entire capsule is 14mm and the diameter of the capsule is 5mm. Find its surface area.



OR

A gulab jamun, contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 gulab jamuns, each shaped like cylinder with two hemispherical ends with length 5cm and diameter 2.8cm.



5

35. The following table gives the distribution of the life time of 400 neon lamps:

Life time (in hours)	Number of lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

Find the average life time of a lamp.

SECTION E

36. **CASE STUDY 1**

India is competitive manufacturing location due to the low cost of manpower and strong technical and engineering capabilities contributing to higher quality production runs. The production of TV sets in a factory increases uniformly by a fixed number every year. It produced 16000 sets in 6th year and 22600 in 9th year.

1) In which year, the production is 29,200 sets?

2) Find the production in the 8^{th} year.

OR

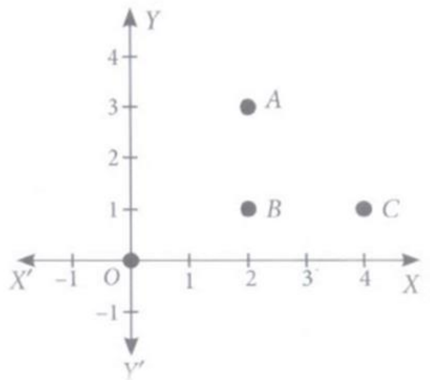
Find the production in first 3 years.

3) Find the difference of the production in 7th year and 4th year.

6

37. **CASE STUDY 2**

Alia and Shagun are friends living on the same street in Patel Nagar. Shagun's house is at the intersection of one street with another street on which there is a library. They both study in the same school and that is not far from Shagun's house. Suppose the school is situated at the point O, i.e., the origin, Alia's house is at A. Shagun's house is at B and library is at C. Based on the above information, answer the following questions.



(i) How far is Alia's house from Shagun's house?

(ii) How far is the library from Shagun's house?

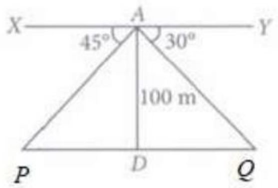
(iii) Show that for Shagun, school is farther compared to Alia's house and library.

OR

Show that Alia's house, shagun's house and library for an isosceles right triangle.

38. **CASE STUDY 3**

A boy is standing on the top of light house. He observed that boat P and boat Q are approaching the light house from opposite directions. He finds that angle of depression of boat P is 45° and angle of depression of boat Q is 30°. He also knows that height of the light house is 100 m.



Based on the above information, answer the following questions.

(i) What is the measure of ∠APD?

(ii) If ∠YAQ = 30°, then ∠AQD is also 30°, Why?

(iii) Find length of PD

OR

Find length of DQ

8

"All the Best & Best of Luck"

****